Edge-State Interferometers in Graphene Nanoribbons

A Time-dependent Modelling

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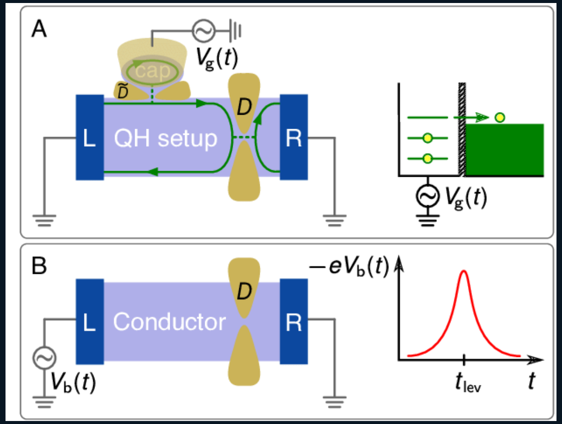
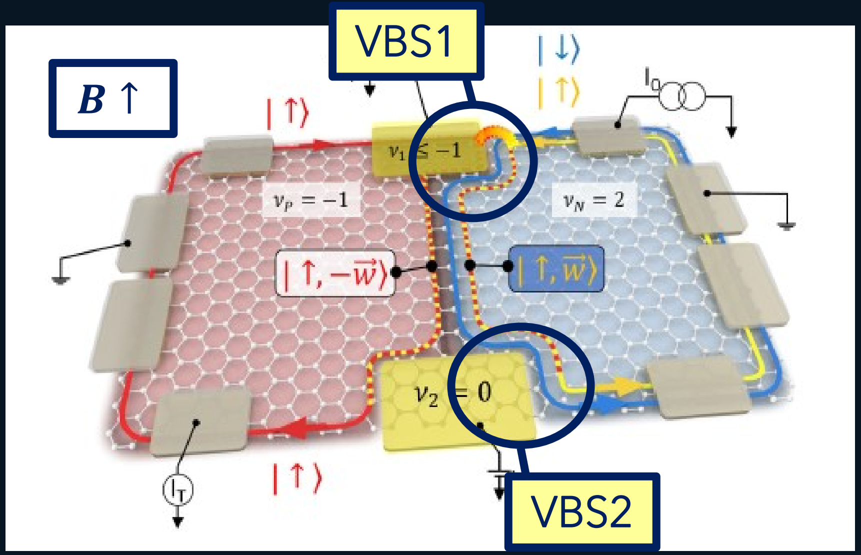
Hall Interferometers for quantum computation

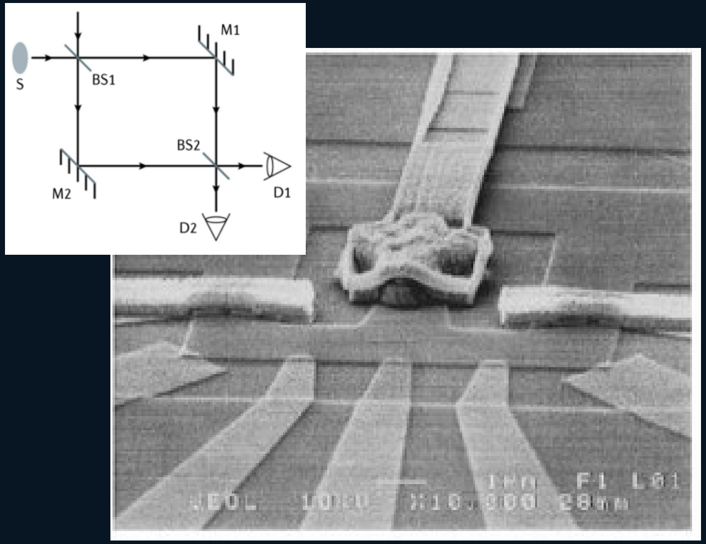
- Integrability into traditional circuitry
- Single Electron Sources for localized carriers
- Chirality: coherence lengths > $10\ \mu m$ ($20\ mK$)
- Graphene: Dirac fermions + Valleytronics



Dashti et al. Physical Review B 100, 035405 (2019)

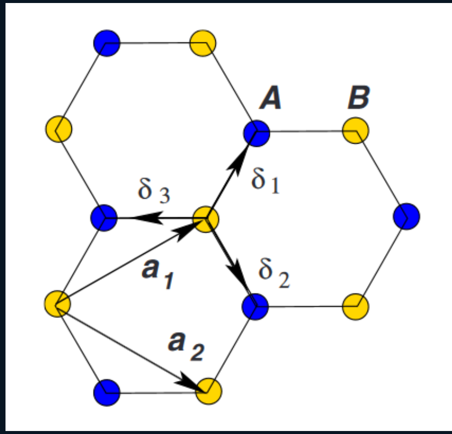


Jo et al. Physical Review Letters 126, 146803 (2021)

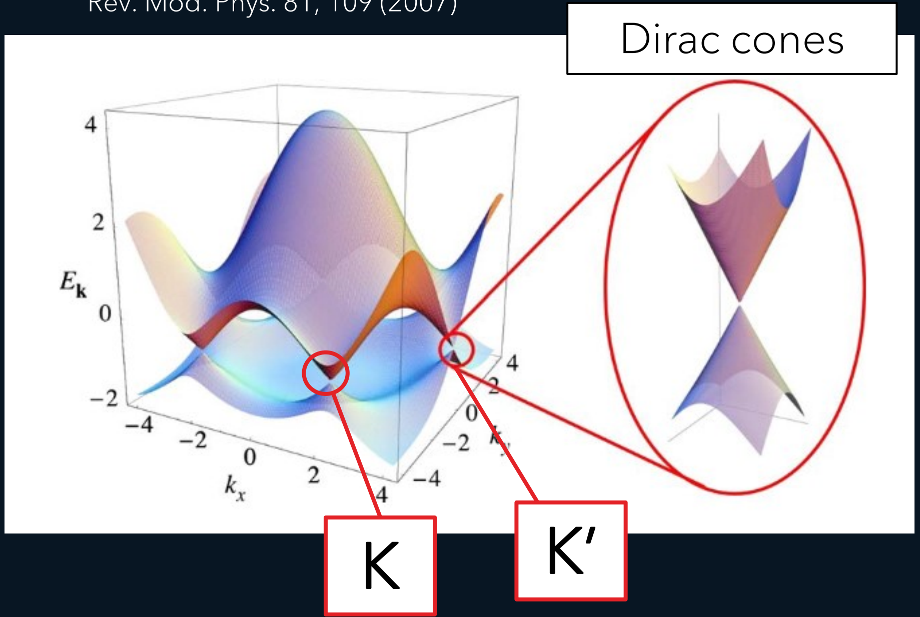


Carrega et al. Nat Rev Phys 3, 698–711 (2021)

Properties of Graphene



Castro Neto et al.
Rev. Mod. Phys. 81, 109 (2007)



Tight-binding Hamiltonian in momentum space:

$$H(k) = -t\sum_\delta \begin{pmatrix} 0 & e^{-ik\cdot\delta} \\ e^{ik\cdot\delta} & 0 \end{pmatrix} + M\sigma_z \approx \qquad q = k - K(K')$$

$$\approx \hbar v_F \begin{pmatrix} M & -q_x + iq_y & & \\ -q_x - iq_y & -M & & 0_{2\times2} \\ & & M & q_x + iq_y \\ 0_{2\times2} & & q_x + iq_y & -M \end{pmatrix}$$

K (upper-left block), K′ (lower-right block)

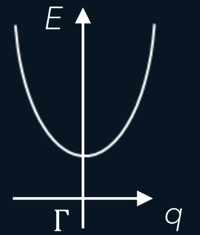
→ *If M=0: Dirac fermions,* $\boldsymbol{v_F} = \frac{\mathbf{3at}}{\mathbf{2\hbar}}$

→ *4-component wave function*

$$\Psi^{K/K'} = (\psi_A^K, \psi_B^K, \psi_A^{K'}, \psi_B^{K'})$$

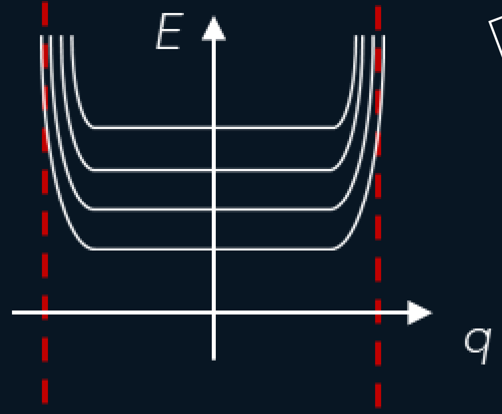
Quantum Hall Effect

GaAs:

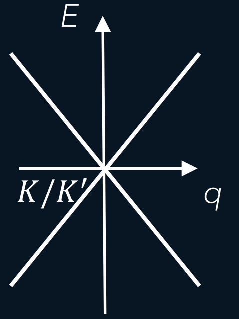


$B \perp$ *2DEG:*

- $q \to q + \frac{eA}{\hbar}, \quad A = (0, x, 0)$
- Bulk: $E_n = E_0 + n\hbar\omega_c$
- Edges: $v_g = \frac{1}{\hbar}\frac{\partial E_n(q)}{\partial q}$

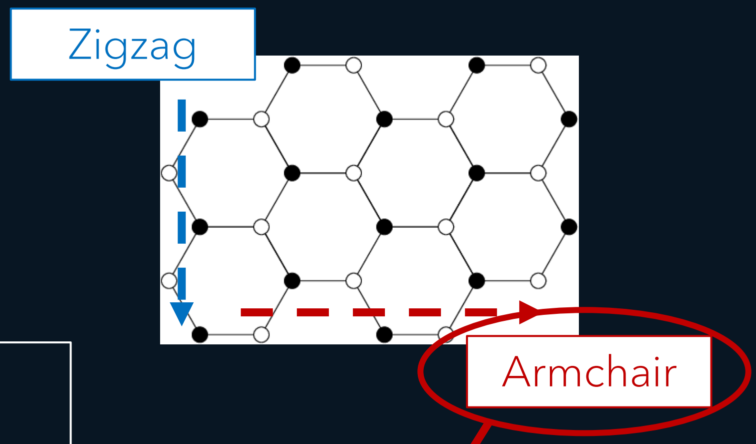


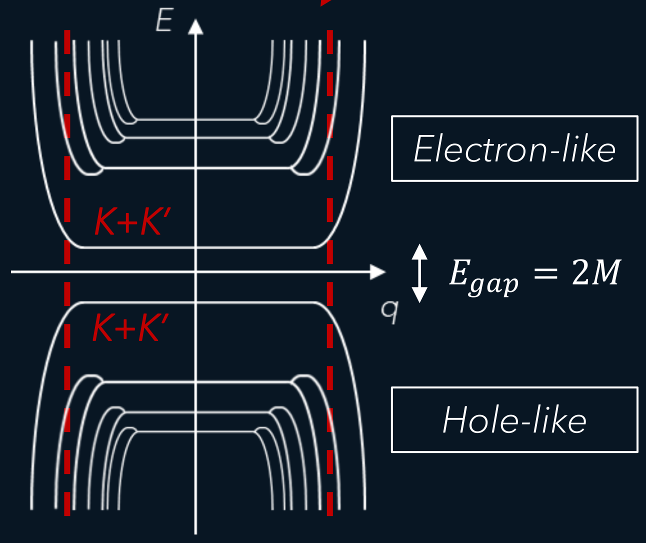
Graphene:



$B \perp$ *Graphene layer:*

- Bulk: $E_n = \pm\sqrt{M^2 + n(\hbar\omega_c)^2}$
  - Valley degenerate
  - Both above and below E_F
- Valley admixing at the edges



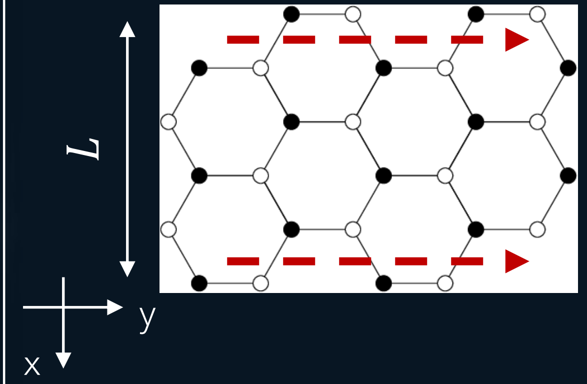


Coherent transport

Valley-dependent evolution

Numerical simulation - Initialization

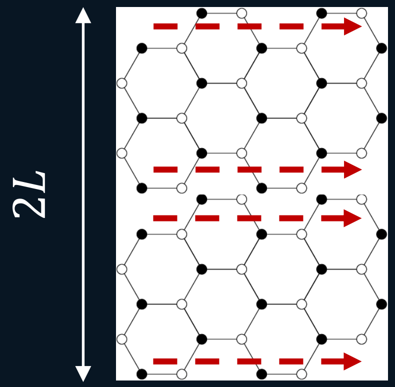
Physical system:



B.C.:

$$\begin{cases} \psi_{A/B}^{K+K'}(0,y) = 0 \\ \psi_{A/B}^{K+K'}(L,y) = 0 \end{cases}$$

Simulated space:



$$\varphi_{A/B}(x,y) = \begin{cases} e^{iK_x x}\psi_{A/B}^{K}(x,y) & 0 \le x \le L \\ e^{iK'_x x}\psi_{A/B}^{K'}(-x,y) & -L \le x \le 0 \end{cases}$$

B.C.: *Continuous + Periodic*

Eigenstate:

$$\varphi_{A/B}^{n,k}(x,y) = \phi_{A/B}^{n}(x,q) \cdot e^{iqy}$$

$$H^{eff}\begin{pmatrix}\phi_A^n \\ \phi_B^n\end{pmatrix} = E\begin{pmatrix}\phi_A^n \\ \phi_B^n\end{pmatrix}$$

Localized on x

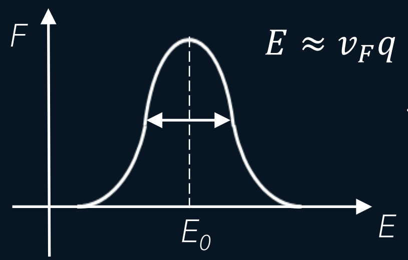
Delocalized on y

Localized Wave-packet:

$$\varphi_{A/B}(x,y) = \frac{1}{\sqrt{2\pi}}\int dq\, F(q)\, \phi_{A/B}^{n}(x,k)e^{iqy}$$

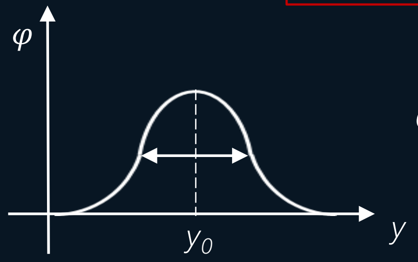
Localized at

$$\left(x_0 = -\frac{\hbar q_0}{eB}, y_0\right)$$



$E \approx v_F q$

$\Longleftrightarrow$



$$\sigma_y \propto \frac{1}{\sigma_E}$$

Numerical simulation - Evolution

$$\hat{H} = \underbrace{V(|x|,y) + M\sigma_z}_{\hat{V}(|x|,y)} - \underbrace{\hbar v_F\left(q_x\sigma_x + q_y\sigma_y\right)}_{\hat{T}(q_x,q_y)} - \underbrace{\frac{\hbar v_F}{l_m^2}|x|\sigma_y}_{\hat{V}_B(|x|)}, \quad l_m = \sqrt{\frac{\hbar}{eB}}$$

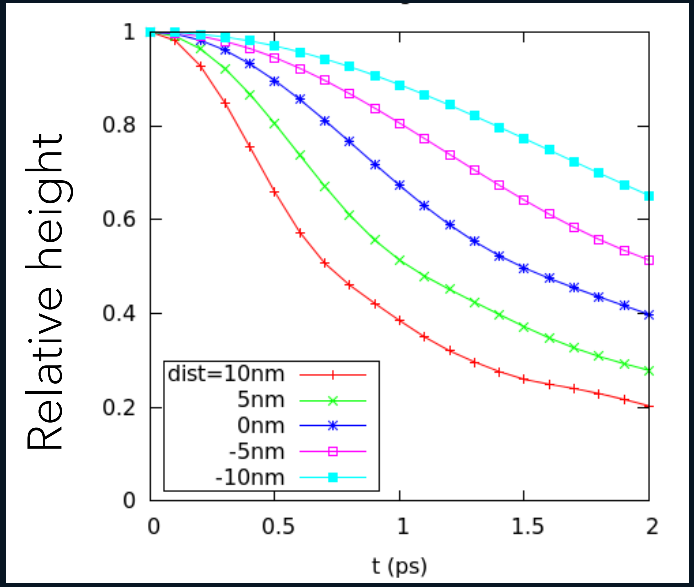
$$\hat{U}(\delta t) = \exp\left(-\frac{i\hat{H}^{eff}\delta t}{\hbar}\right) \approx$$

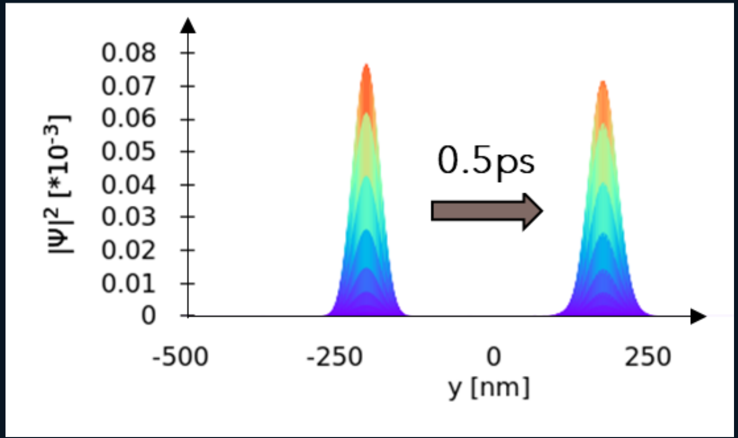
$$\approx F^{-1}\underbrace{\exp\left(-\frac{i\hat{T}\delta t}{\hbar}\right)}_{\textit{Momentum space}} F \underbrace{\exp\left(-\frac{i\hat{V}_B\delta t}{\hbar}\right)\exp\left(-\frac{i\hat{V}\delta t}{\hbar}\right)}_{\textit{Real space}}$$

Trotter-Suzuki factorization
+ Split-Step Fourier method

Evolution couples the sublattice degrees of freedom

$$\begin{pmatrix}\varphi_A(N\delta t)\\ \varphi_B(N\delta t)\end{pmatrix} = \left[\hat{U}(\delta t)\right]^N \begin{pmatrix}\varphi_A(0)\\ \varphi_B(0)\end{pmatrix}$$

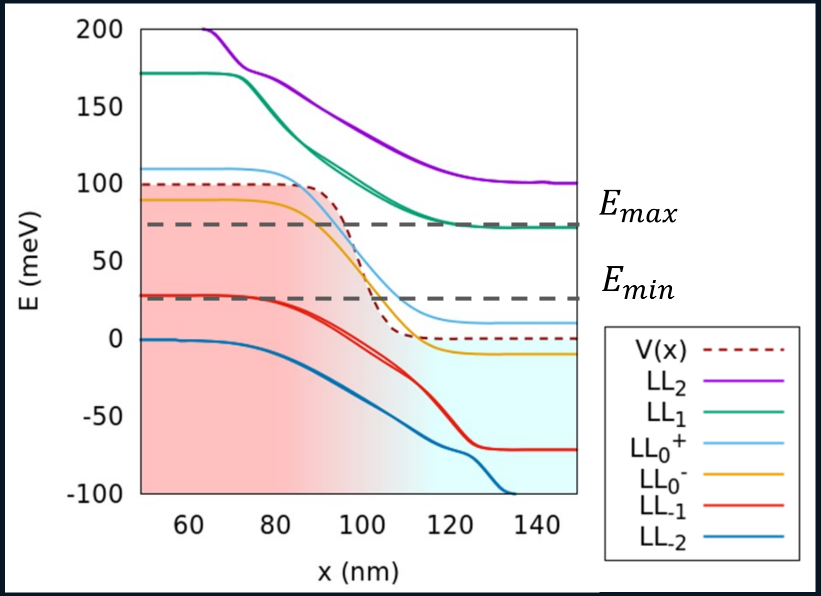




Moving along junctions

$\sigma_y = 30\text{ nm} \rightarrow \sigma_E \approx 7.63\text{ meV} \quad (6\sigma_E \approx 45.78\text{ meV})$

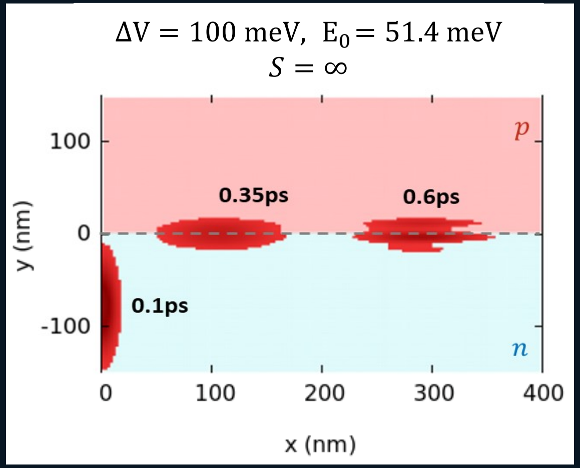
$$V(x) = \Delta V \frac{1}{e^{-Sx}+1}$$



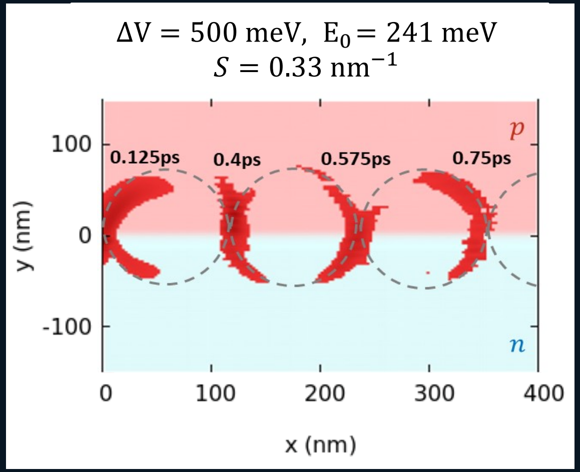
Forghieri et al., Phys. Rev. B 106, 165402 (2022)

- *Interchannel mixing: el-like + hole-like states*
  - *Cyclotron radius:* $r_c = \frac{|p|}{|q|B} \approx \frac{|E_0 - V|}{v_F|q|B}$

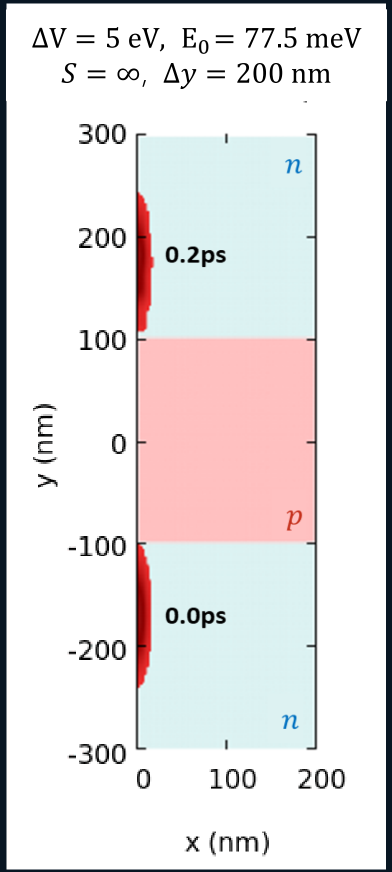
1) Edge States → $l_m \gg r_c$



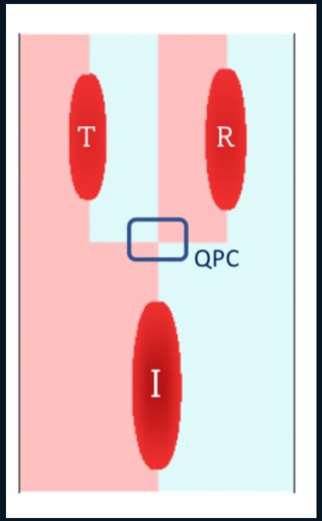
2) Snake States → $r_c \gg l_m$



3) Klein tunneling

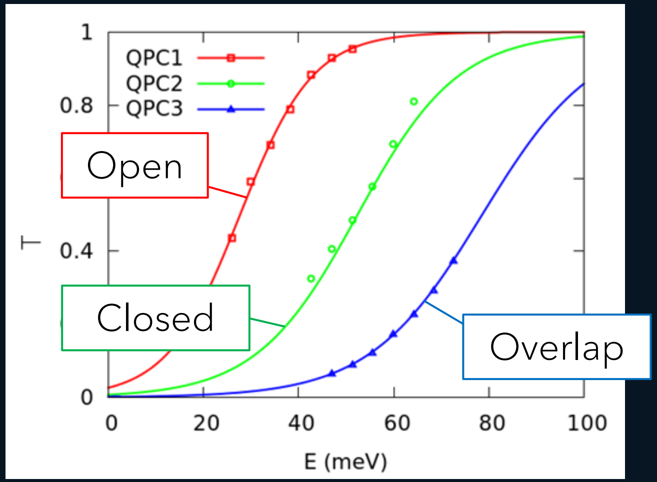


MZI with Quantum Point Contacts

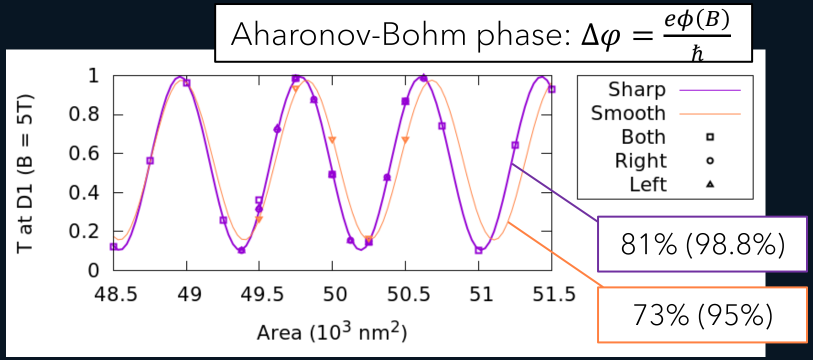


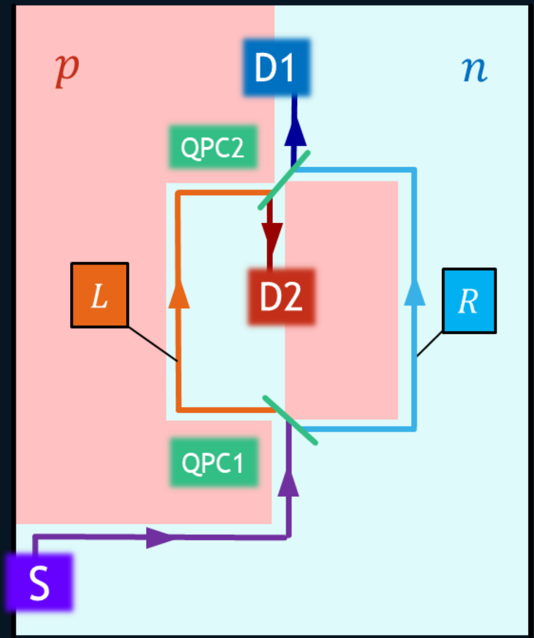
$$T(E) = 1 - R(E) = \frac{1}{e^{-\alpha(E-E_{QPC})}+1}$$

- Channels with same chirality
- Can transmit *below* the barrier



- High visibility (> GaAs)
- Localization → Phase averaging





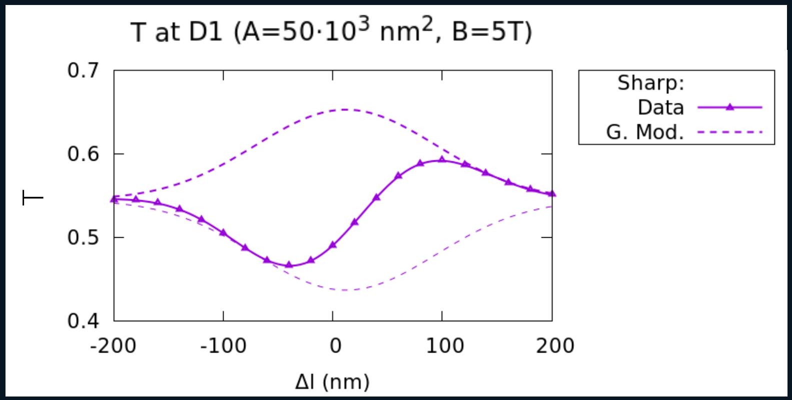
Dephasing

$$\Delta\theta = \frac{2\pi\Delta l}{L}$$

+

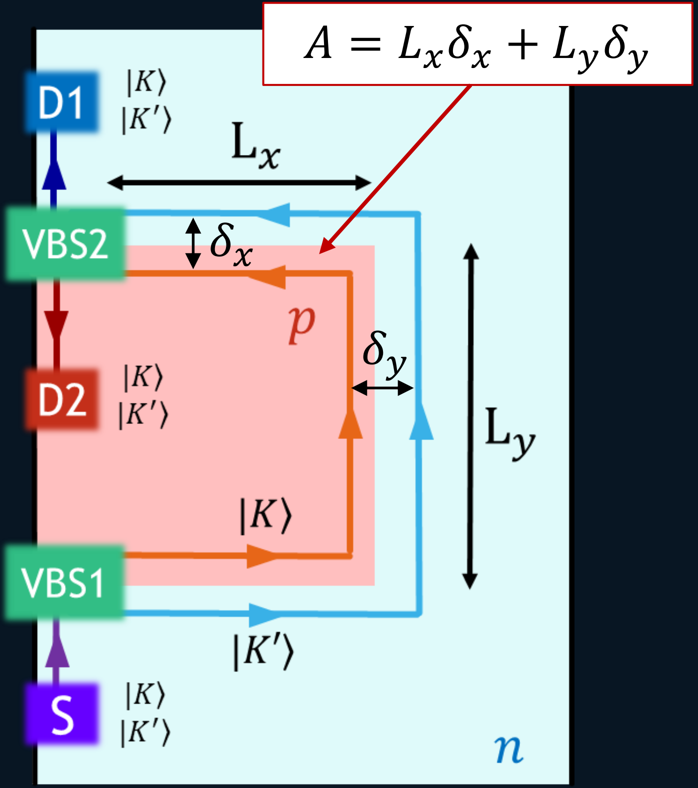
Averaging

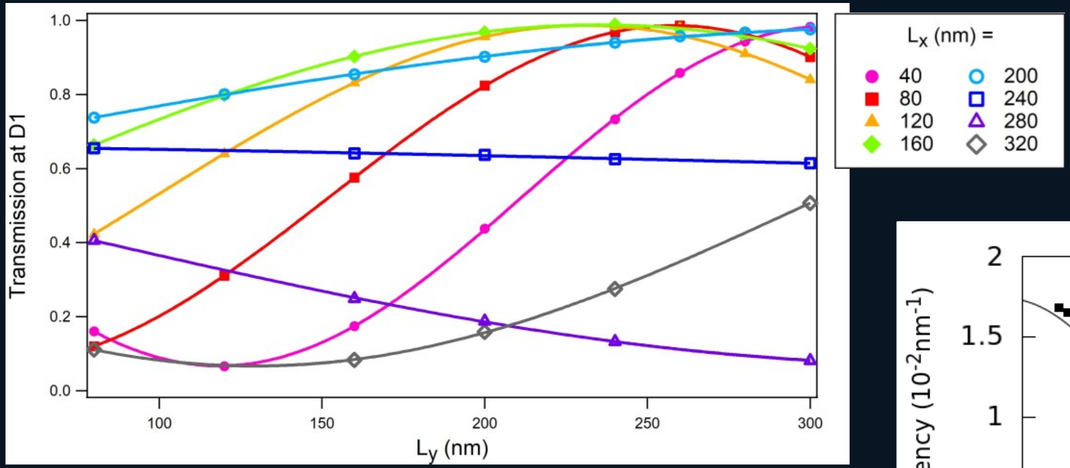
$$F^{avg}(\Delta l) = (F^R * F^L)(\Delta l) = \propto e^{-\Delta l^2/4(\sigma_R^2+\sigma_L^2)}$$



MZI with Valley Beam Splitters

$M \neq 0$:
- Equilibration along edges
- T, R only depend on M, S





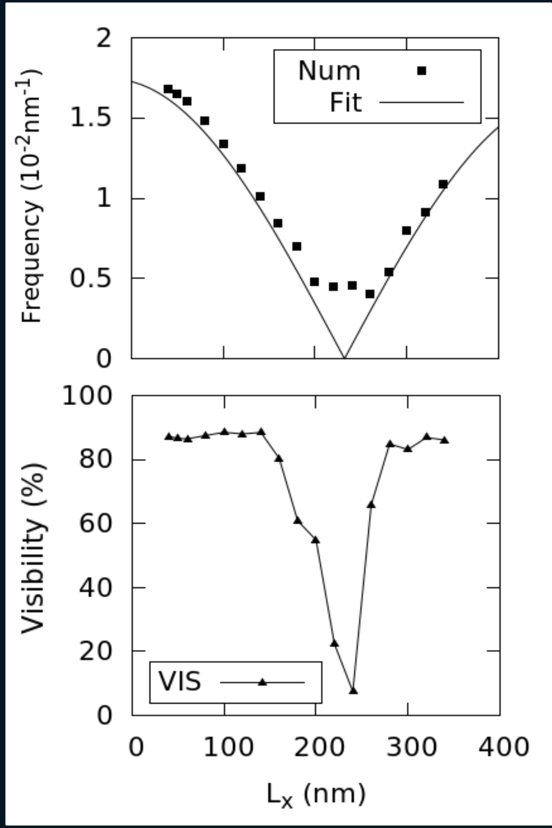
- Minimal phase averaging
- Varing L_y: $f = eB|\delta_y|/\hbar$
- Frequency changes with L_x

⇩

Non-trivial channel shape

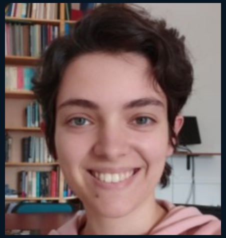
+

Drop in visibility



Future perspectives

- Detailed characterization of VBS
- Analysis of recombination processes along channels
- Two-particle interferometry
- Longer simulations with HPC resources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