Weak Values: a new paradigm to characterize nanoscale systems

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0 – Traditional Theory-Experiment Link

I – Weak Value Theory-Experiment Link

II – Unprecedented Characterization

(a) Closed system **Quantum Thermalization**

(b) Expected current in THz electronics

(c) The **cut-off frequency** in **nanoscale transistors**

(d) Quantum **Work** and **two-time correlations**

III – Non-Markovian Stochastic Schrödinger Equations

(a) Unravelling Non-Markovian open quantum systems

(b) Quantum electron transport with **Monte Carlo trajectories**

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THEORY

$$\langle \psi | \hat{A} | \psi \rangle$$

Bra-ket of observable A (with operator $\hat{A}$) for $|\psi\rangle$

EXPERIMENT

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$$\langle\psi|\hat{A}|\psi\rangle$$

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EXPERIMENT

1. Prepare $|\psi(t)\rangle$
2. Measure **A strongly**
3. Get **average of A** measurements

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Expected A for $|\psi\rangle$

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THEORY **EXPERIMENT**

Uncertainty of A

$$\langle\psi|\hat{A}^2|\psi\rangle - \langle\psi|\hat{A}|\psi\rangle \quad = \quad (\Delta A)^2$$

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Cross-correlation between **A,B**

$$\langle\psi(t_2)|\hat{B}\hat{U}_{t_1}^{t_2}\hat{A}|\psi(t_1)\rangle \; = \; \mathbb{E}[A(t_1)B(t_2)]$$

$$if\ \hat{A},\hat{B}\ commute$$

I – Weak Value
Theory-Experiment Link

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$$\frac{\langle b_k|\hat{A}|\psi(t)\rangle}{\langle b_k|\psi(t)\rangle}$$

Weak value of A on $|\psi\rangle$, if $|b_k\rangle$

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Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. 60, 1351 (1988).

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- Is a **complex** number
- $|b_k\rangle$ **eigenstate** of $\hat{B} = \sum_k b_k |b_k\rangle\langle b_k|$
- **Recover expectation**

$$\mathbb{E}[A] = \langle\psi|\hat{A}|\psi\rangle = \sum_k |\langle b_k|\psi\rangle|^2 \, {}_{\langle b_k|}A_{|\psi\rangle}$$

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3. Measure **B strongly**

↓

4. Get **average of A** using **only cases when** found $\boldsymbol{b_k}$ **for B**

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$$\mathbb{E}\left[A_{weak}|B = b_k\right]$$

Conditional expectation
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$$Re\left\{\frac{\langle b_k|\hat{A}|\psi(t)\rangle}{\langle b_k|\psi(t)\rangle}\right\} = \mathbb{E}[A_{weak}|B = b_k]$$

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THEORY

EXPERIMENT

$\boldsymbol{Im}$ **if** a_{weak} is **weak ancilla's momentum**

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- **Im and Re** parts using **weak post-selected** measurement **(expectations)**

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  D. Pandey et al., Phys. Rev. A 103, 052219 (2021).

- **Im and Re** parts using **strong post-selected** measurement **(expectations)**

  E. Cohen and E. Pollak, Phys. Rev. A 98, 042112 (2018).

  F. De Zela. Phys. Rev. A 105.4, 042202 (2022)

Measuring the **wavefunction** $\psi(\vec{x}, t)$

$$c\,\psi(\vec{x}, t) = \frac{\langle \vec{p} = 0 | \vec{x} \rangle \langle \vec{x} | \psi(t) \rangle}{\langle \vec{p} = 0 | \psi(t) \rangle}$$

I – Weak Value Theory-Experiment Link

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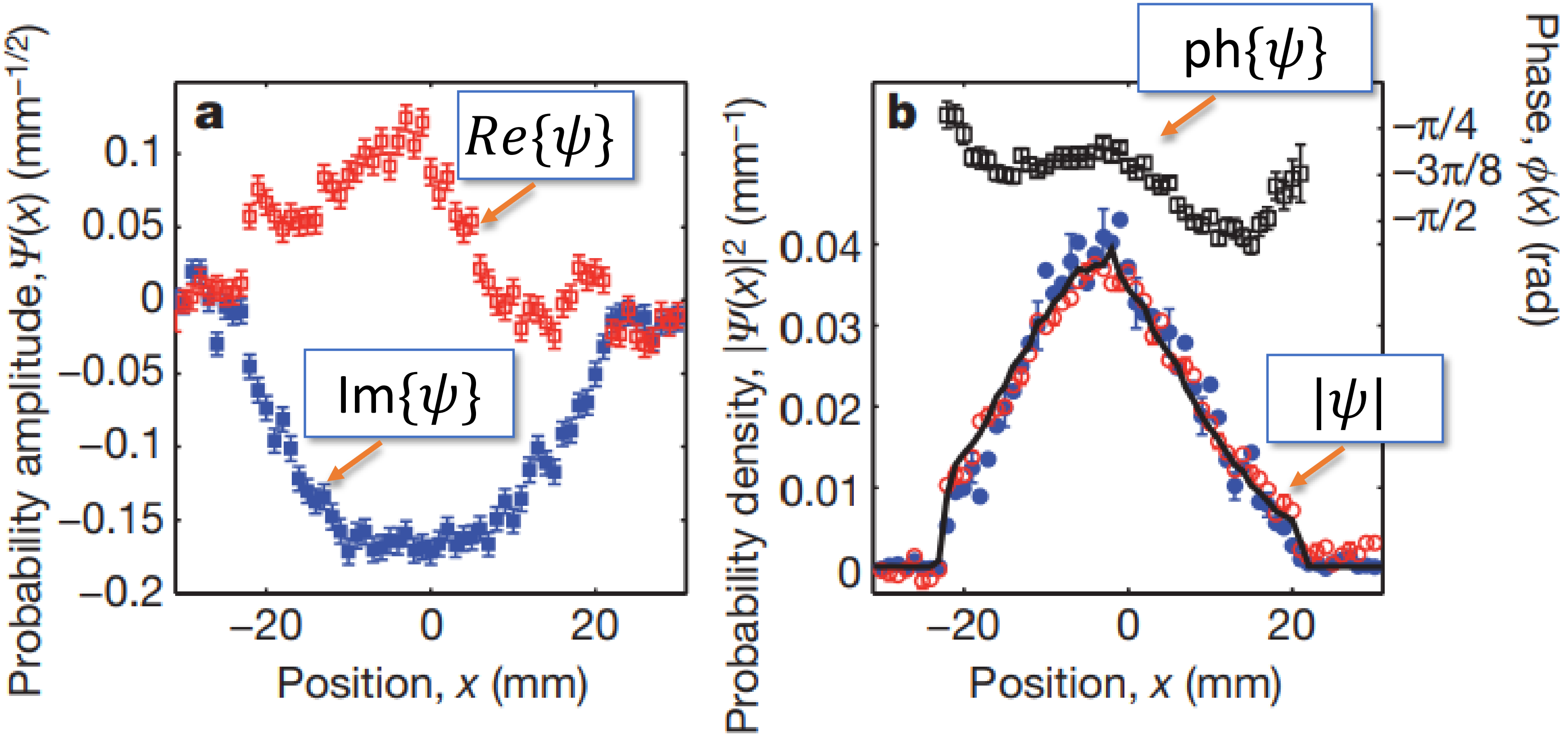
J. S. Lundeen, B. Sutherland, A. Patel, C. Stewart, and C. Bambe,
***Nature** 474, 188 (2011)*



I – Weak Value Theory-Experiment Link

Quantum Particle **Trajectories**?

$$\vec{\nabla} S(\vec{x},t) = Re\left\{ \frac{\langle \vec{x} \left| \hat{\vec{p}} \right| \psi(t) \rangle}{\langle \vec{x} | \psi(t) \rangle} \right\}$$

$$\psi(\vec{x},t) = R(\vec{x},t)\ e^{\frac{i}{\hbar}S(\vec{x},t)}$$

polar form

S. Kocsis, B. Braverman, S. Ravets, M. J. Stevens, R. P. Mirin, L. K. Shalm, and A. M. Steinberg.
***Science** 332, no. 6034 (2011): 1170-1173.*

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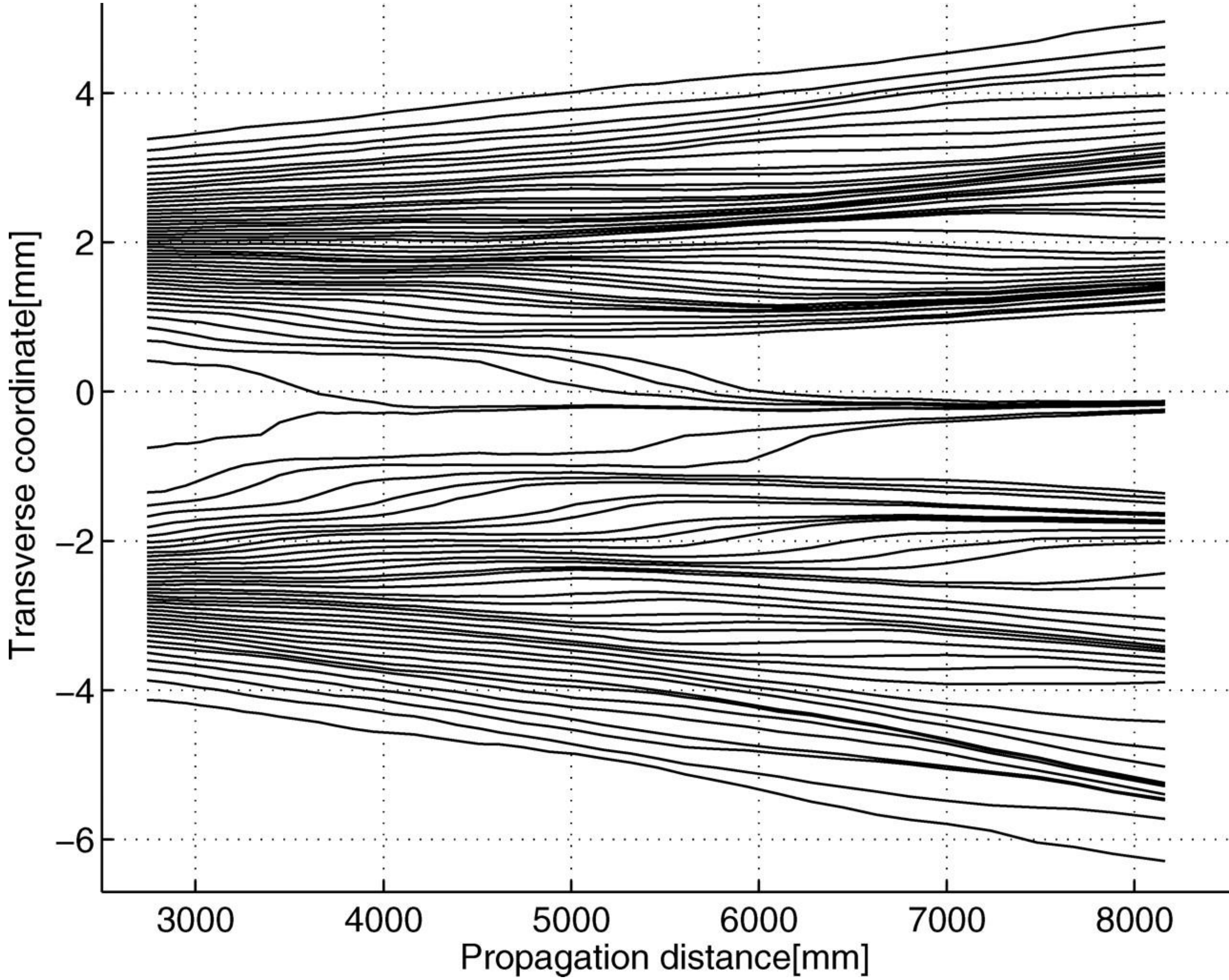

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II – Unprecedented Characterization

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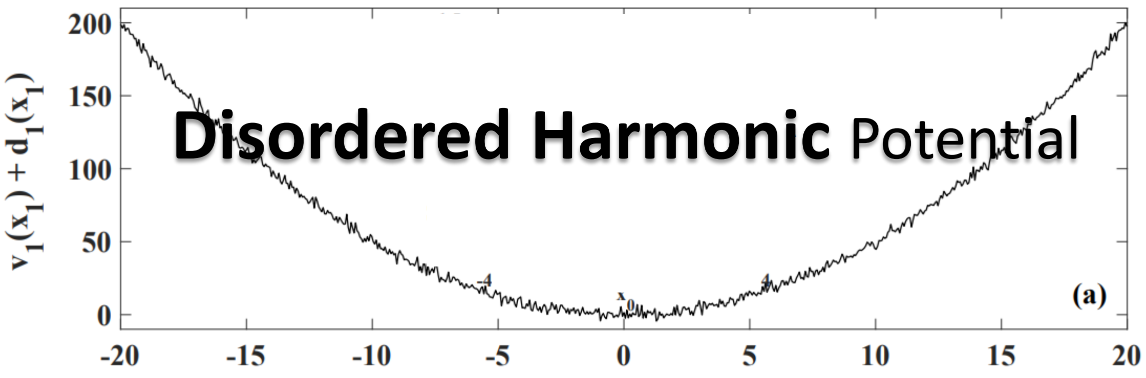

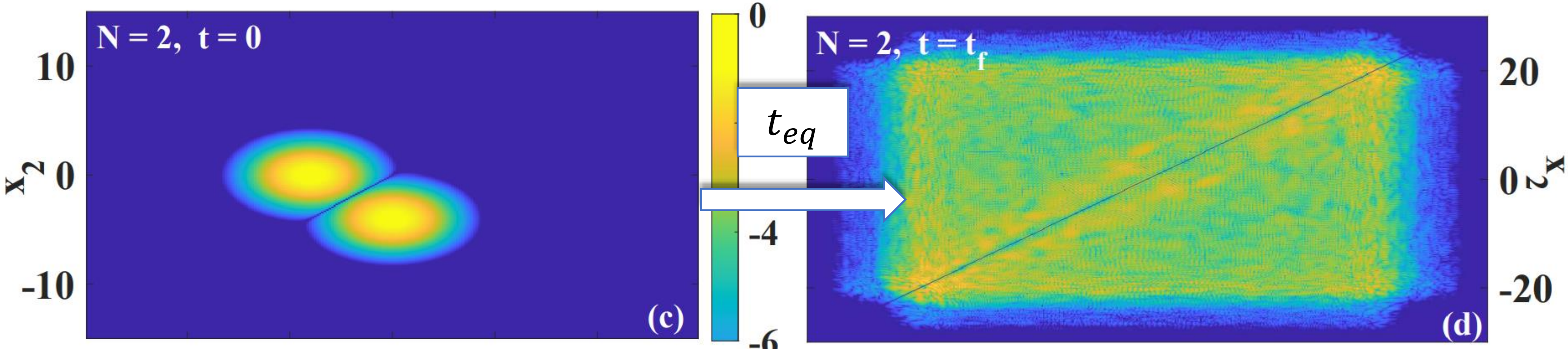

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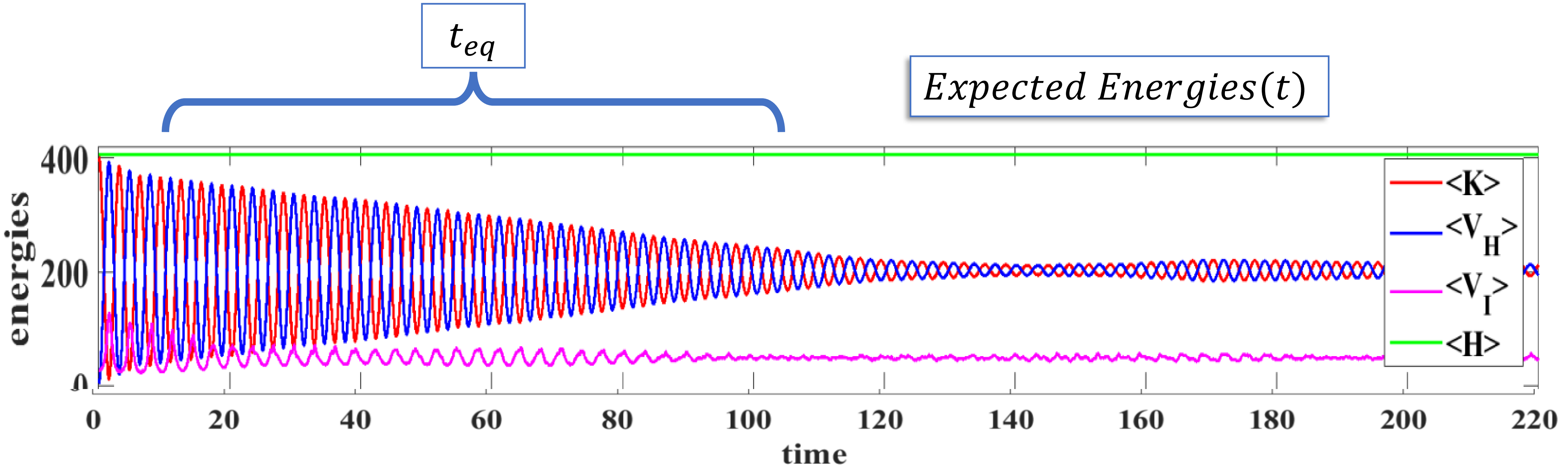

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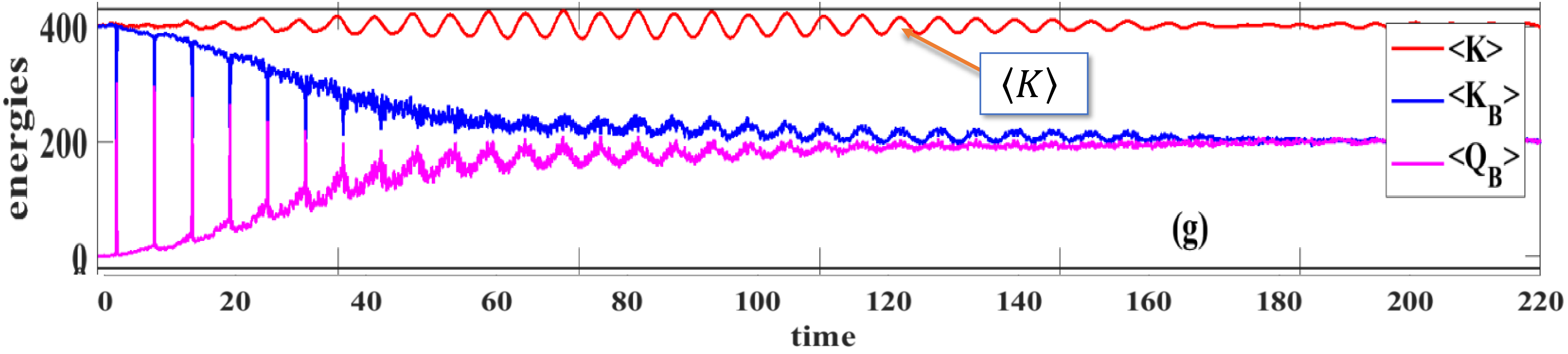
$$\langle K \rangle = \langle K_B \rangle + \langle K_O \rangle$$



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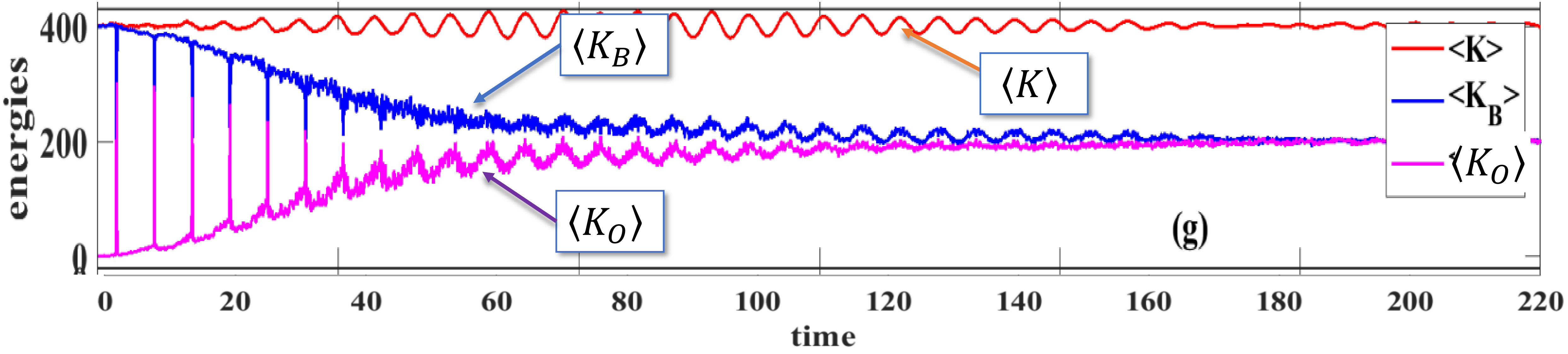

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Trajectory formulations as heuristic tools?

$$g^{\psi}(\vec{x},t) = Re\left\{ \frac{\langle \vec{x}|\hat{G}|\psi(t)\rangle}{\langle \vec{x}|\psi(t)\rangle} \right\}$$

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- $\mathcal{E}^{\psi}(\vec{x},t) = Re\left\{ \frac{\langle \vec{x}|\hat{\boldsymbol{H}}|\psi(t)\rangle}{\langle \vec{x}|\psi(t)\rangle} \right\} = \frac{\mathscr{p}^{\psi}(\vec{x},t)^2}{2m} + V(\vec{x},t) + Q(\vec{x},t)$ Bohm Energy

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- $\ell_z^{\psi}(\vec{x},t) = Re\left\{ \frac{\langle \vec{x}|\hat{\boldsymbol{L}}_{\boldsymbol{z}}|\psi(t)\rangle}{\langle \vec{x}|\psi(t)\rangle} \right\} = x_1 p_2(\vec{x},t) - x_2 p_1(\vec{x},t)$ — Bohm Angular Momentum

…

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(b) Expected current in THz electronics

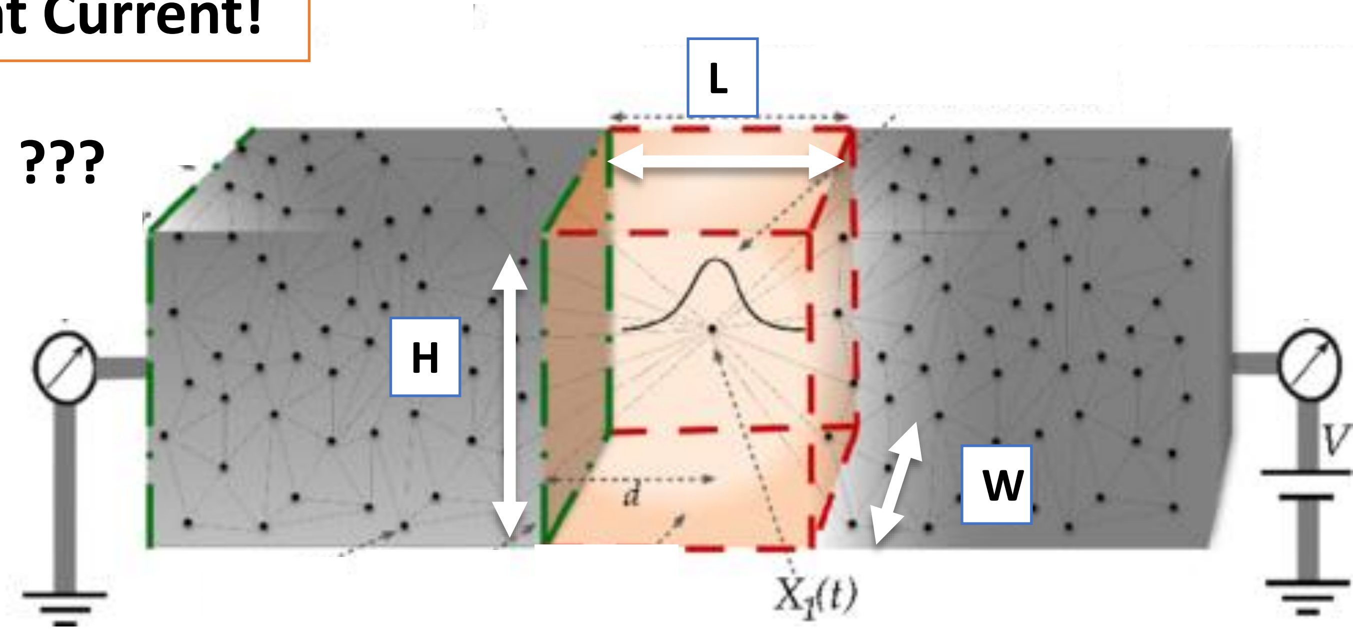
$$I_k^{\xi}(t) = \int_{\Omega} \vec{J}_k^{\xi}(\vec{r},t) \cdot d\vec{s} + \int_{\Omega} \varepsilon(\vec{r},t) \frac{\partial \vec{E}_k^{\xi}(\vec{r},t)}{\partial t} \cdot d\vec{s}$$

Particle Current

Displacement Current!

Operator: $\hat{J}$

Operator: ???



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Quantum RSP theorem → $L \ll H, W$

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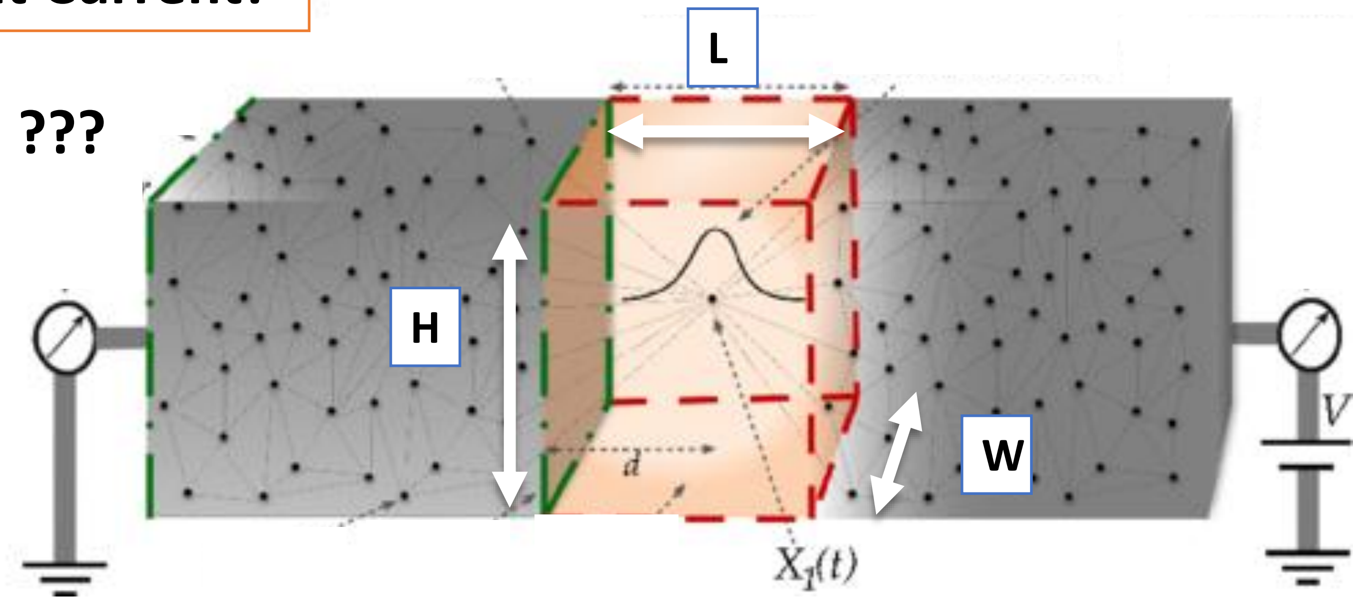

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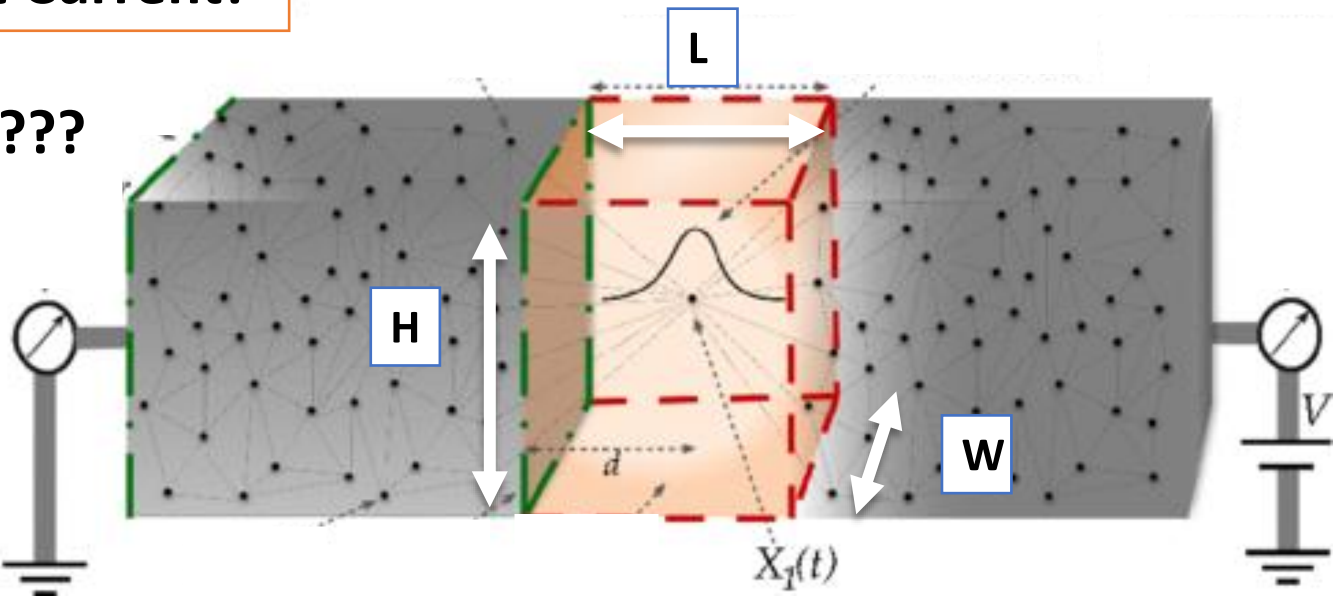
$$\mathbb{E}[I_T](t) = \lim_{|\sigma|\to\infty} \frac{1}{|\sigma|}\sum_{\xi\in\sigma}\sum_{k=1}^{N} I_k^{\xi}(t)$$



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(c) The cut-off frequency in nanoscale transistors

Consensus dwell time

$$\tau_D := \int_0^\infty dt \int_\Gamma |\psi(\vec{r},t)|^2 dr$$

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Average Trajectory Dwell-Time

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$\sigma^* \equiv$ ***Only*** *those* ***entering*** *the barrier*

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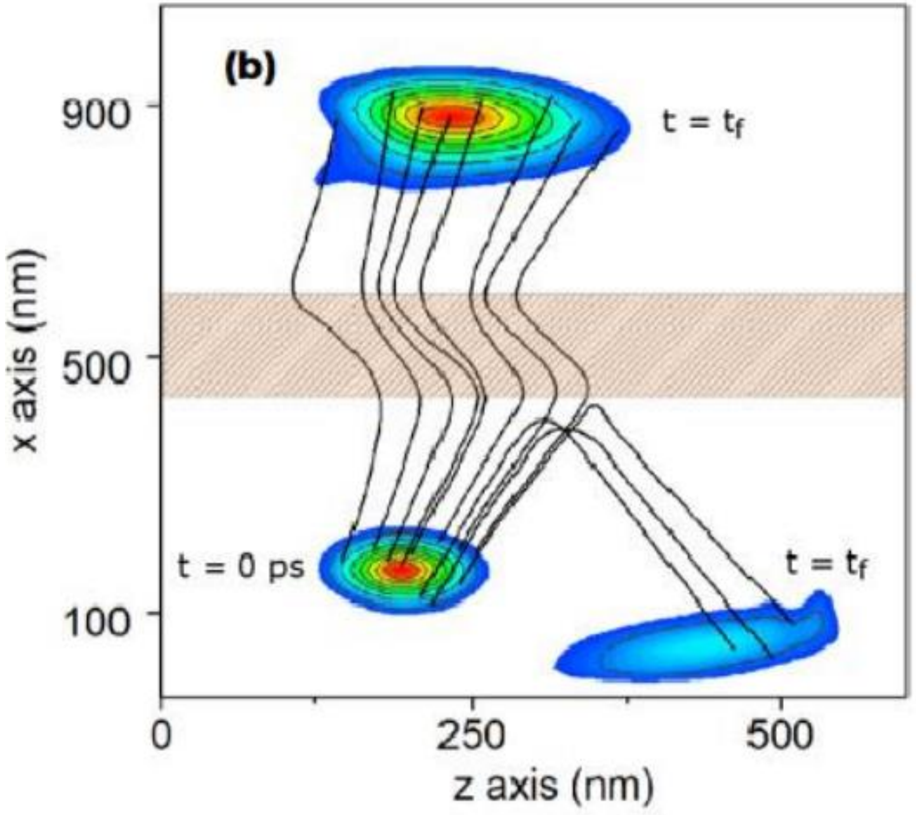

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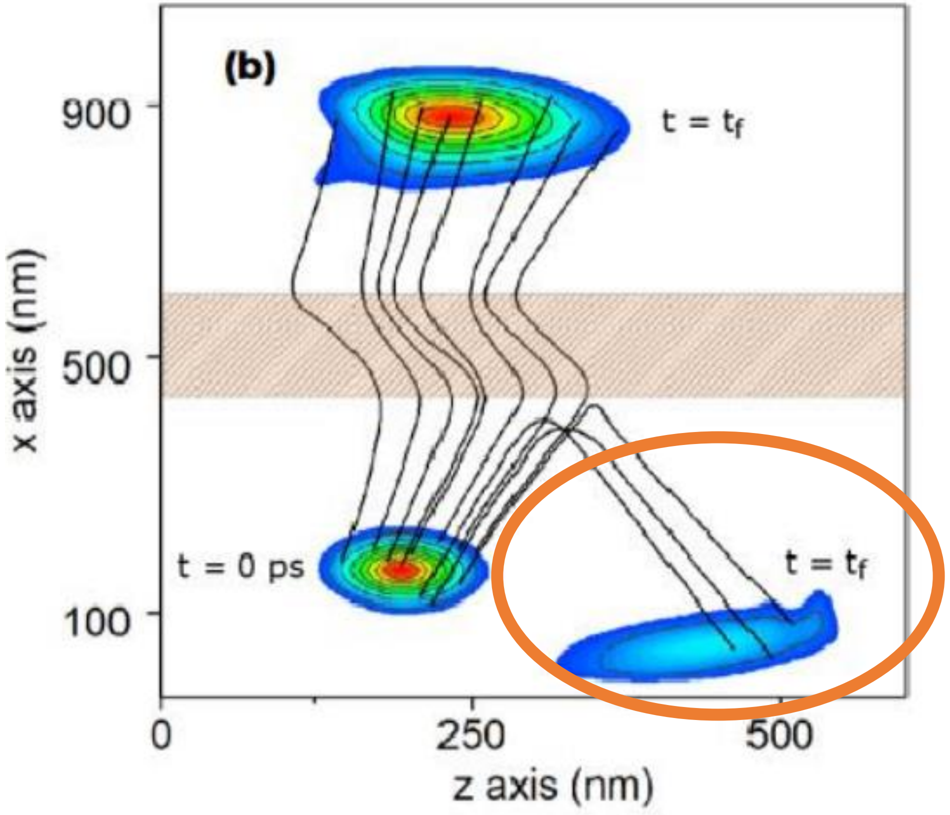

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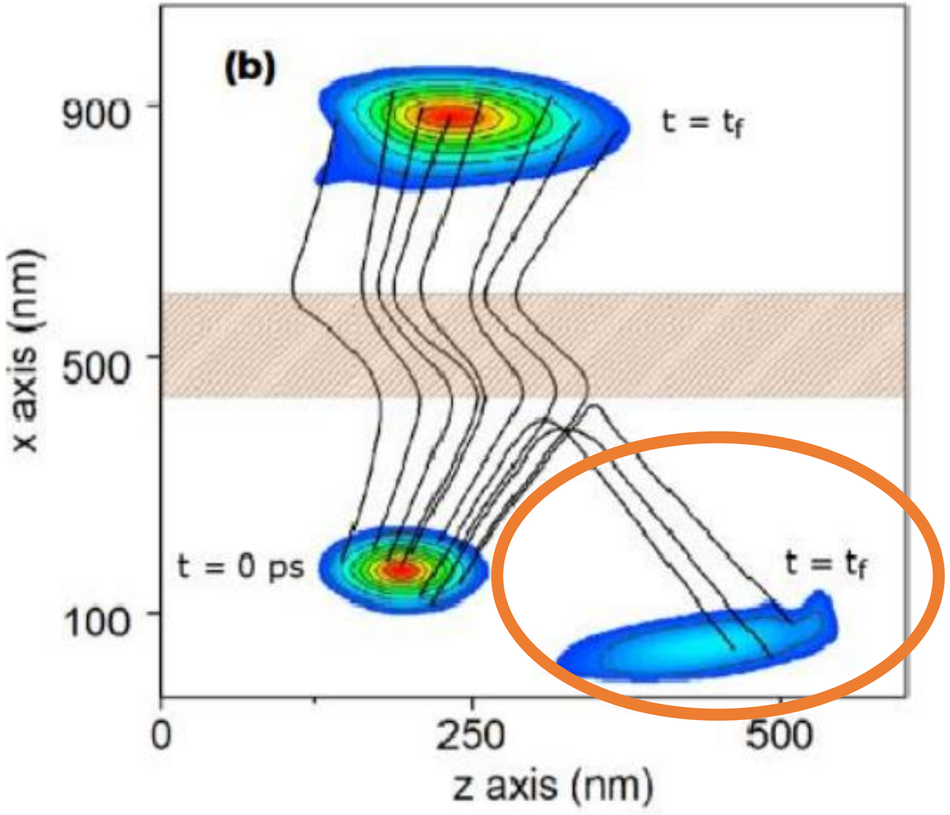
More informative Cut-off Frequency?

$$f_B = \frac{1}{\tau_{DB}}$$



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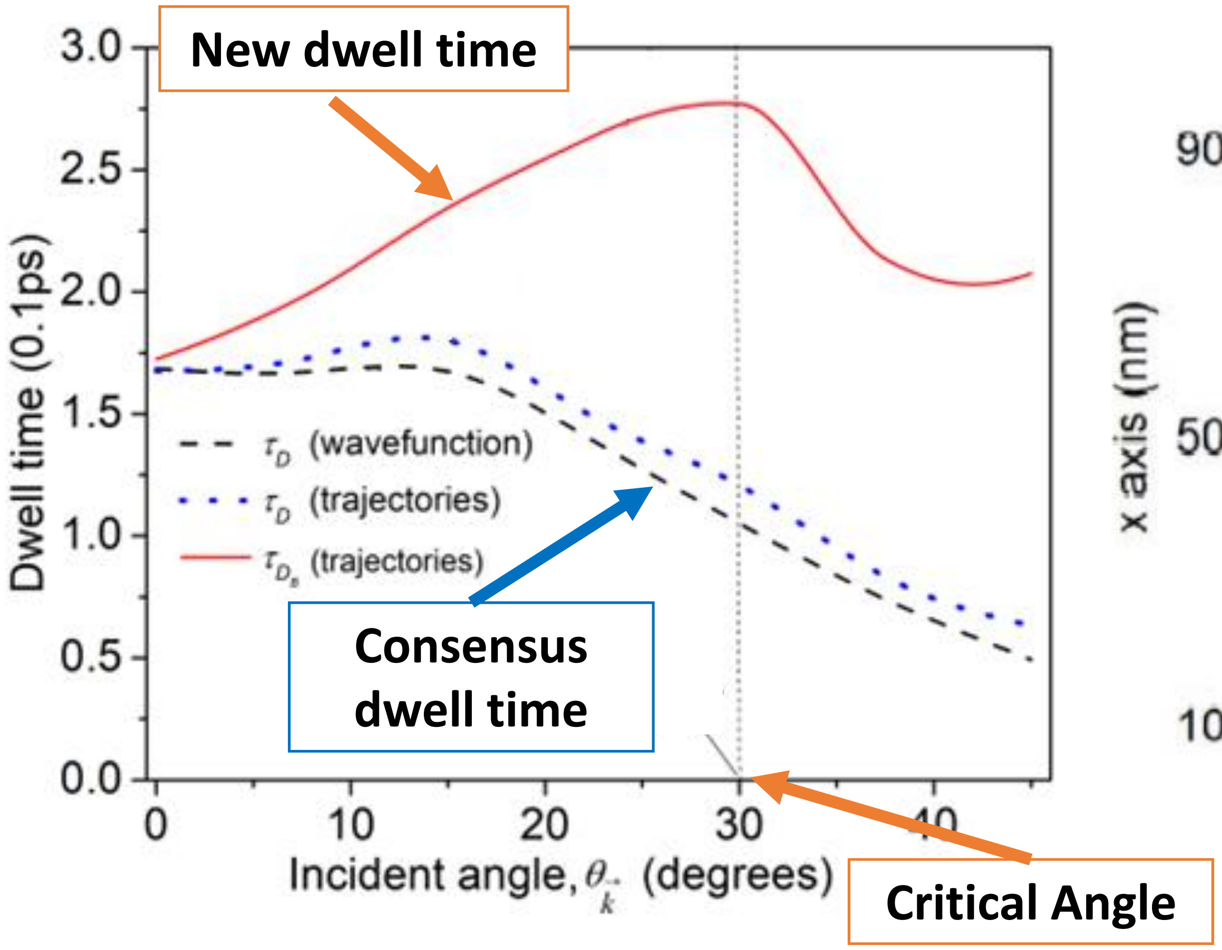
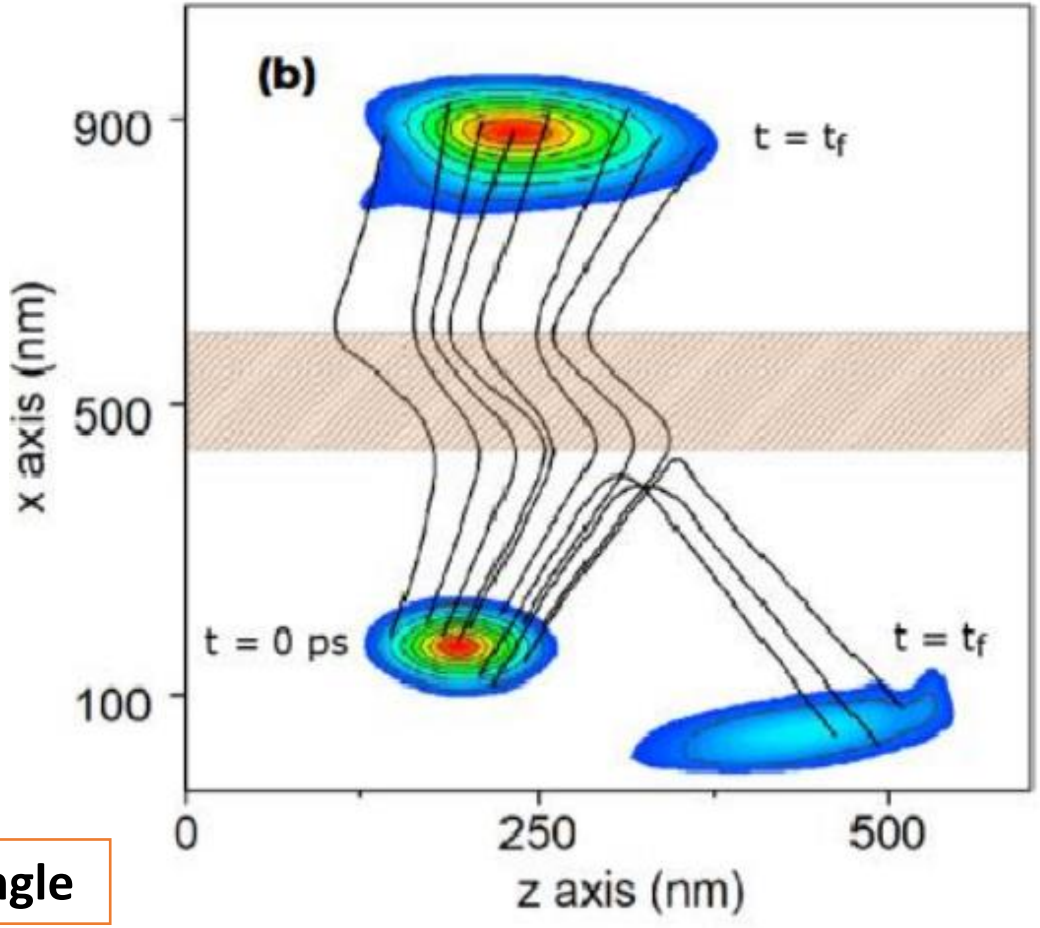
(c) The cut-off frequency in nanoscale transistors





D. Pandey, M.Villani, E. Colomes, Z. Zhan and X.Oriols, Sci. Technol. 34, 034002 (2019)

III – A Recipe for non-Markovian Open Quantum Systems

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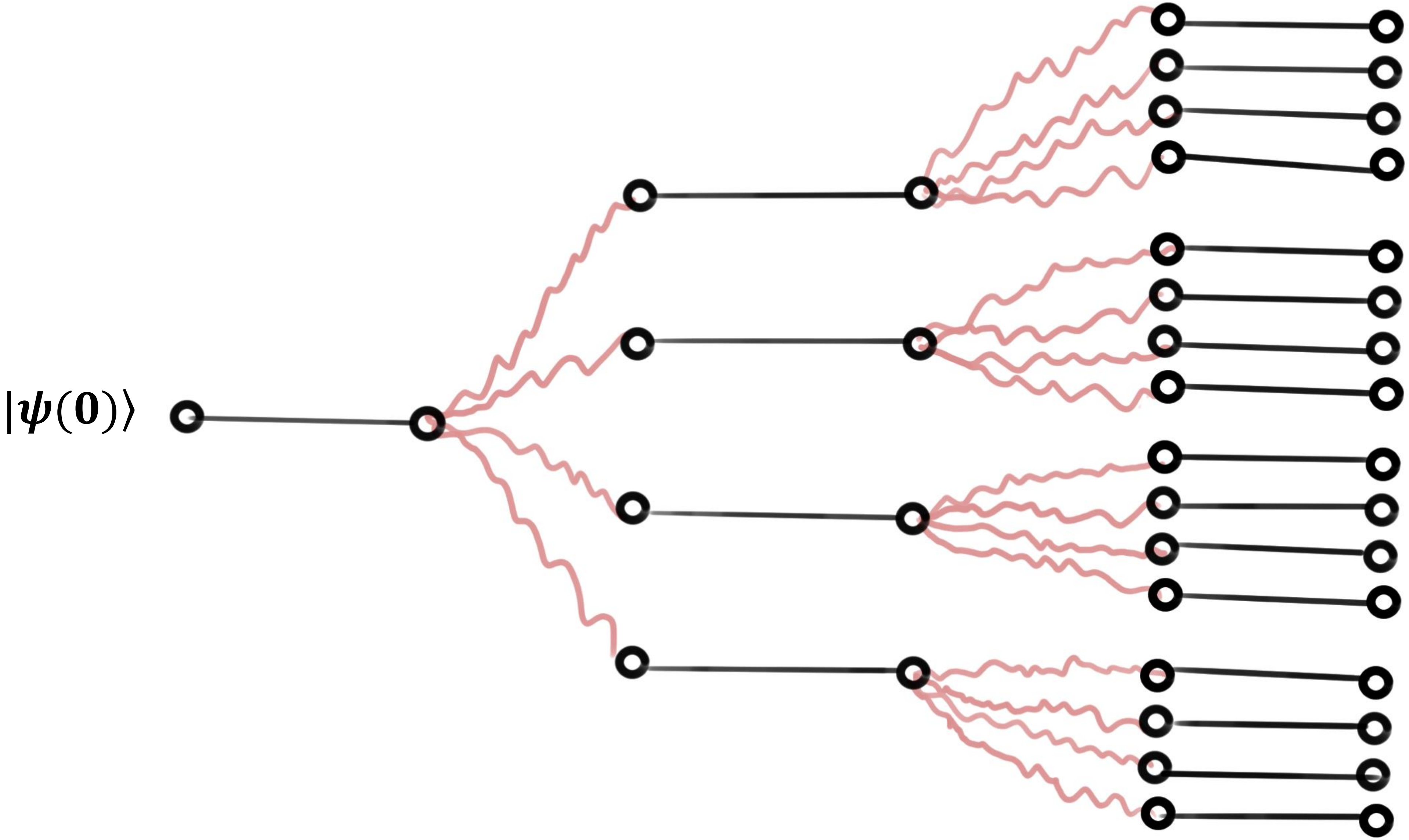
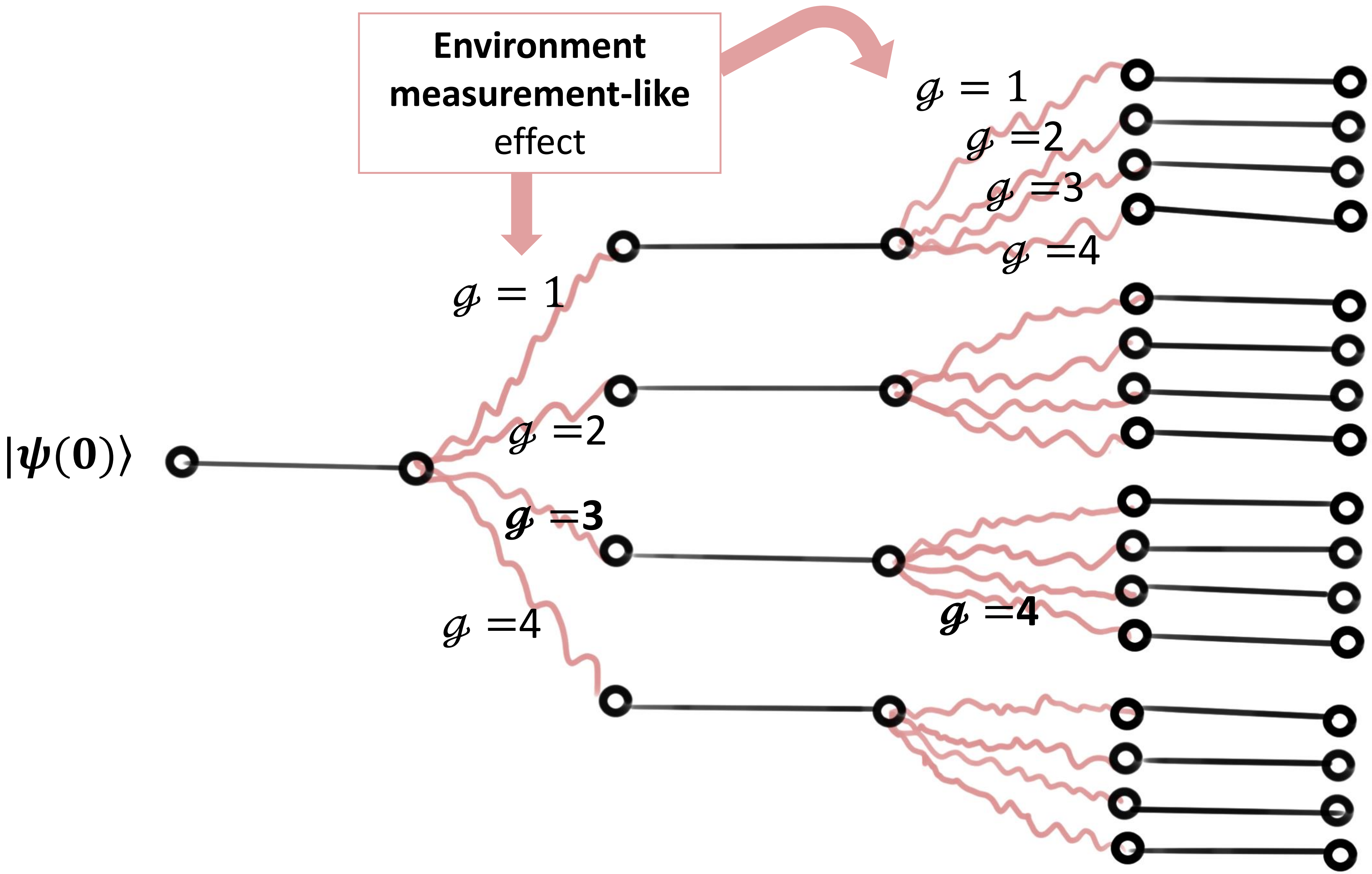


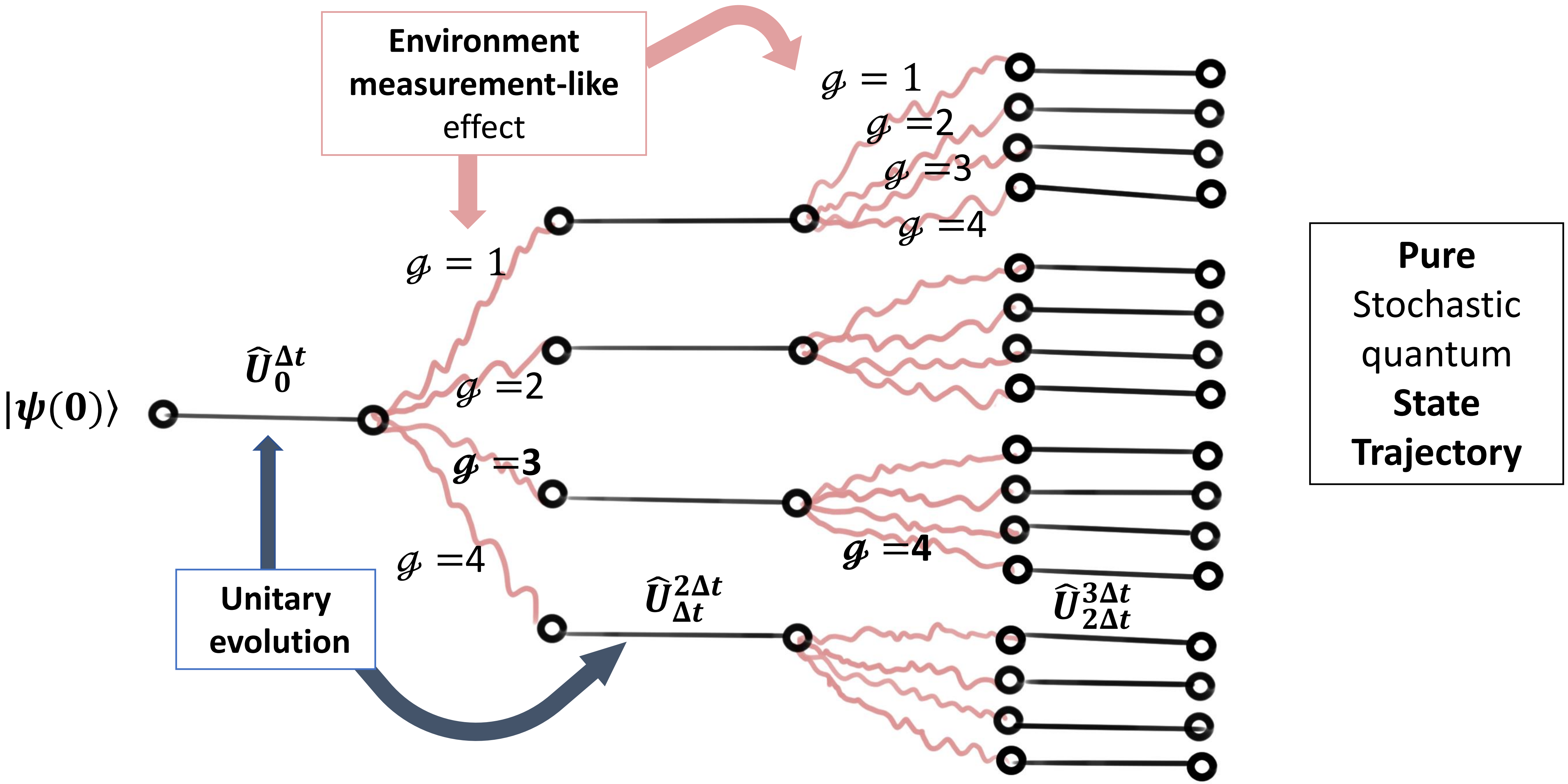
Pure
Stochastic
quantum
State
Trajectory

III – A Recipe for non-Markovian Open Quantum Systems



Pure Stochastic quantum **State Trajectory**

III – A Recipe for non-Markovian Open Quantum Systems



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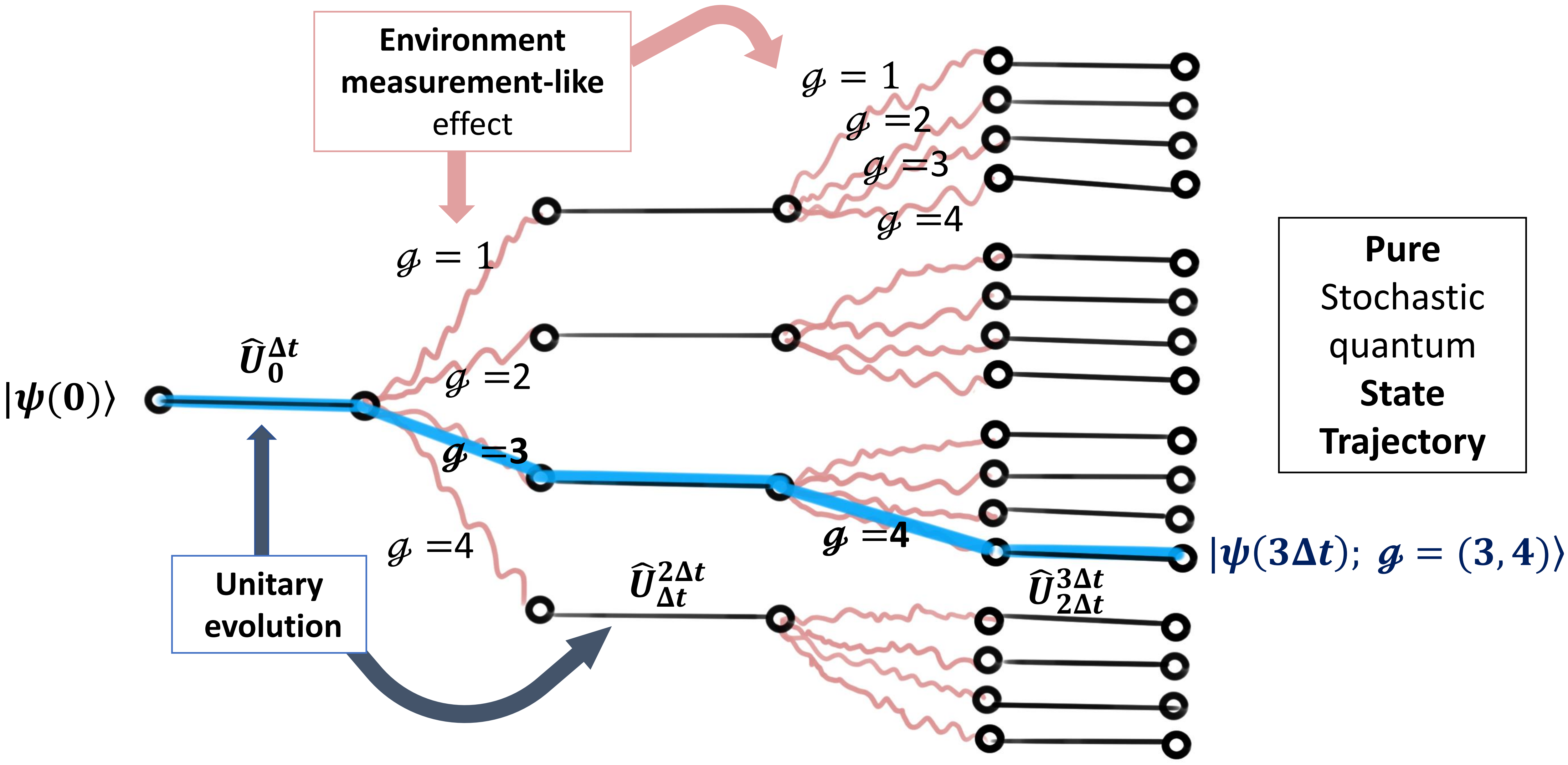

III – A Recipe for non-Markovian Open Quantum Systems

(a) Markovian open quantum systems

Markovian Master Equation for **reduced density matrix**

⟷

Markovian Stochastic Schrödinger Equation unravelling some $\widehat{\mathrm{G}}$

$$\hat{\rho}_S(t) := tr_E[\hat{\rho}_{SE}] = \mathbb{E}_\xi\left[\,|\psi(t); g^\xi(t)\rangle\langle\psi(t); g^\xi(t)|\,\right]$$

K. Jacobs and D. A. Steck, Contemp. Phys. 47, 279 (2006)

L. Li, M. J. Hall, and H. M. Wiseman, Phys. Rep. 759, 1 (2018)

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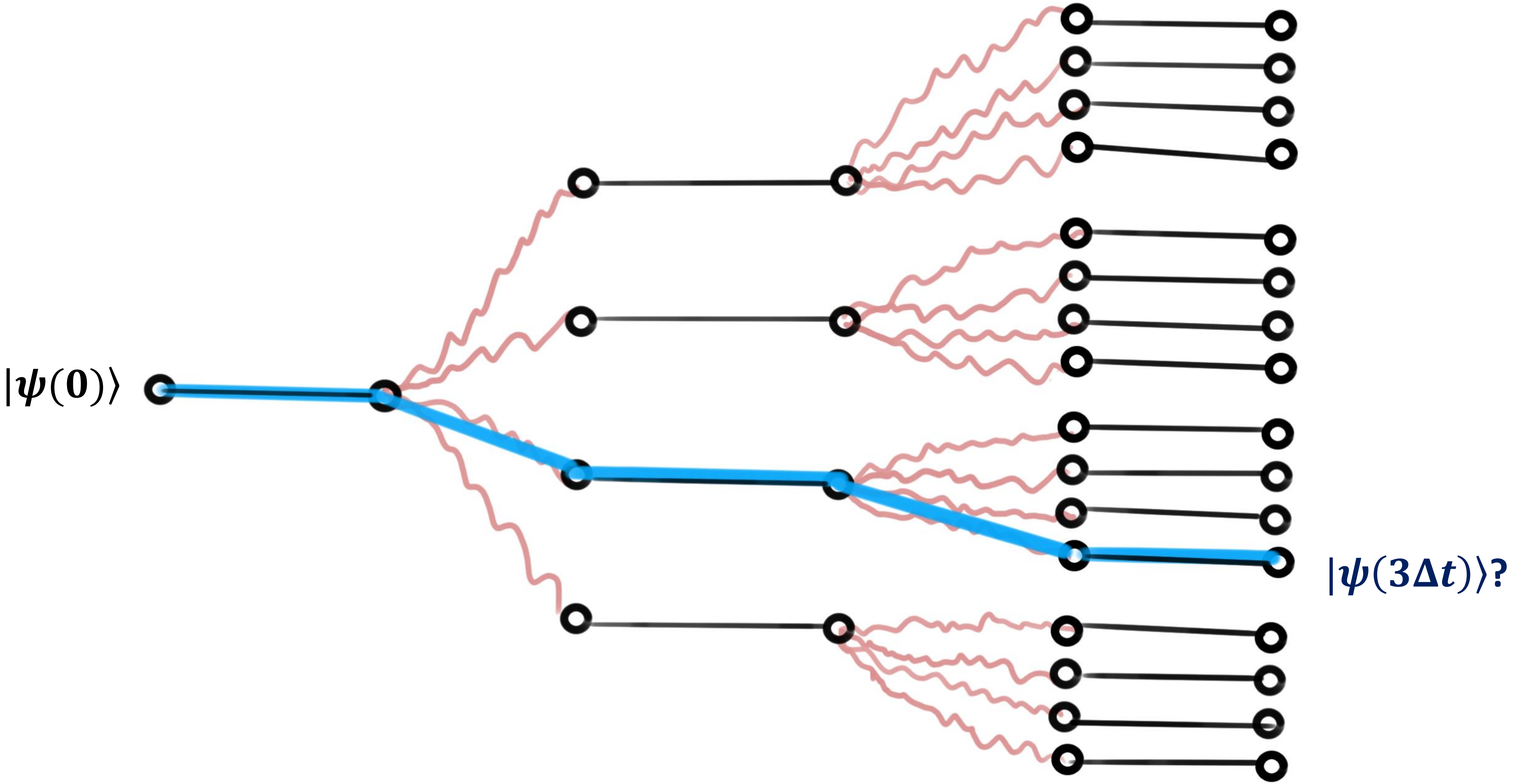
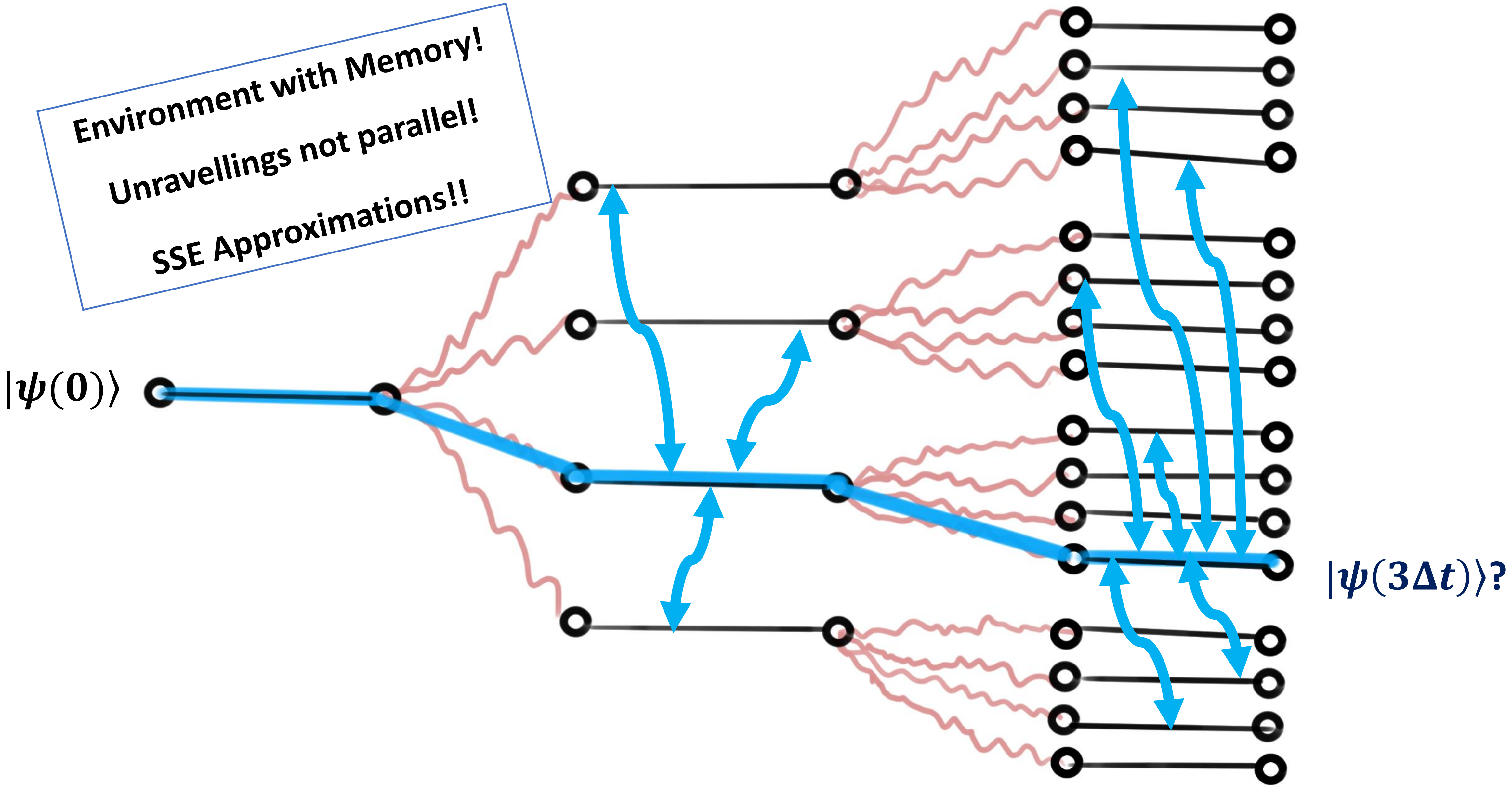
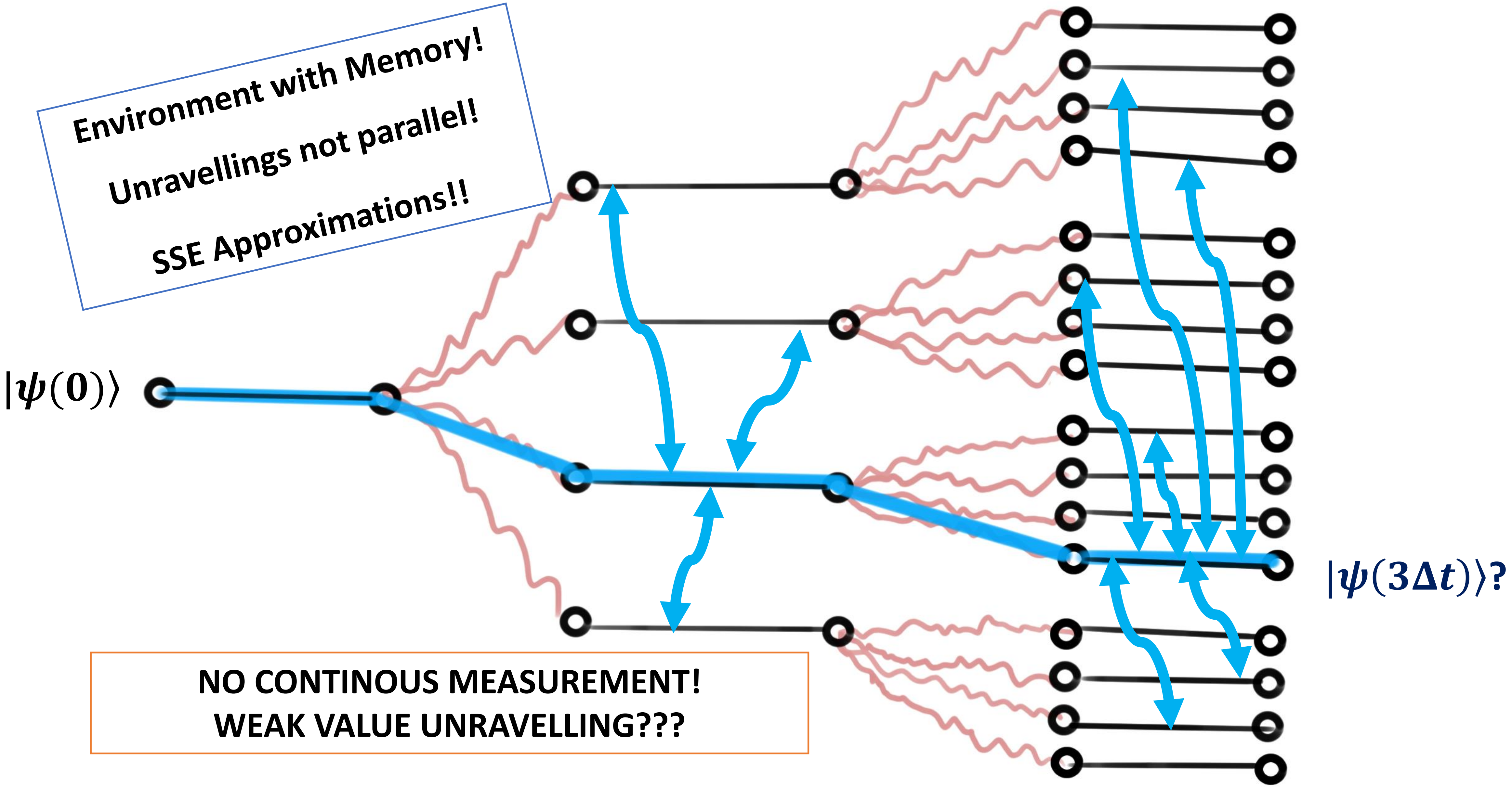


III – A Recipe for non-Markovian Open Quantum Systems



III – A Recipe for non-Markovian Open Quantum Systems



NO CONTINOUS MEASUREMENT!
WEAK VALUE UNRAVELLING???

III – A Recipe for non-Markovian Open Quantum Systems

(b) SSEs Unravelling Weak Values

$$g^{\xi}(t) \rightarrow \overrightarrow{x_E}^{\xi}(\mathrm{t})$$

$$\left\{ i\hbar \frac{\partial \psi^{\xi}(\overrightarrow{x_S}, t)}{\partial t} = \left[-\sum_{j \in S} \frac{\hbar^2}{2m_j} \frac{\partial^2}{\partial x_j^2} + U\left(\overrightarrow{x_S}, \overrightarrow{x_E}^{\xi}(t), t\right) + \mathcal{P}\left(\overrightarrow{x_S}, \overrightarrow{x_E}^{\xi}(t), t\right) \right] \psi^{\xi}(\overrightarrow{x_S}, t) \right.$$

H. M. Wiseman and J. M. Gambetta, Phys. Rev. A 68, 062104 (2003) & Phys. Rev. Lett. 101, 140401 (2008)

X. Oianguren-Asua, CF. Destefani , M. Villani, DK. Ferry, X. Oriols. Chapter of: Physics and the Nature of Reality (2023)

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$$\mathcal{P}(\overrightarrow{x_S},\overrightarrow{x_E},t) := \sum_{k\in E}\left[-\frac{1}{2}m_k v_k(\vec{x},t) + Q_k(\vec{x},t) - i\frac{\hbar}{2}\frac{\partial v_k(\vec{x},t)}{\partial x_k}\right]$$

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In non-Markovian,
only approximately
knowable!

H. M. Wiseman and J. M. Gambetta, Phys. Rev. A 68, 062104 (2003) & Phys. Rev. Lett. 101, 140401 (2008)

X. Oianguren-Asua, CF. Destefani , M. Villani, DK. Ferry, X. Oriols. Chapter of: Physics and the Nature of Reality (2023)

III – A Recipe for non-Markovian Open Quantum Systems

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In non-Markovian,
only approximately
knowable!

But**, even in non-Markovian** **so Almost SSE !**

$$\hat{\rho}_S(t) = \mathbb{E}_{\xi}\left[\, |\psi^{\xi}(t); g_{\xi}(t)\rangle \langle \psi^{\xi}(t); g_{\xi}(t)|\, \right]$$

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In non-Markovian, only approximately knowable!

But, **even in non-Markovian** **so Almost SSE !**

$$\hat{\rho}_S(t) = \mathbb{E}_{\xi}\left[|\psi^{\xi}(t); g_{\xi}(t)\rangle \langle \psi^{\xi}(t); g_{\xi}(t)| \right]$$

➡ **Guide approximation** by classical intuition

H. M. Wiseman and J. M. Gambetta, Phys. Rev. A 68, 062104 (2003) & Phys. Rev. Lett. 101, 140401 (2008)

X. Oianguren-Asua, CF. Destefani , M. Villani, DK. Ferry, X. Oriols. Chapter of: Physics and the Nature of Reality (2023)

(c) Quantum electron transport with Monte Carlo trajectories

$$I_{Total}^{\xi}(t) = \sum_{k=1}^{N} I_k^{\xi}(t) \Rightarrow$$

Two-time **correlations** (PSD), **noise** distribution, **logical operation frequency** etc.

D. Marian, N. Zanghì, and X. Oriols, Phys. Rev. Lett. 116, 110404 (2016)
D. Pandey, E. Colomés, G. Albareda, and X. Oriols, Entropy 21, 1148 (2019)

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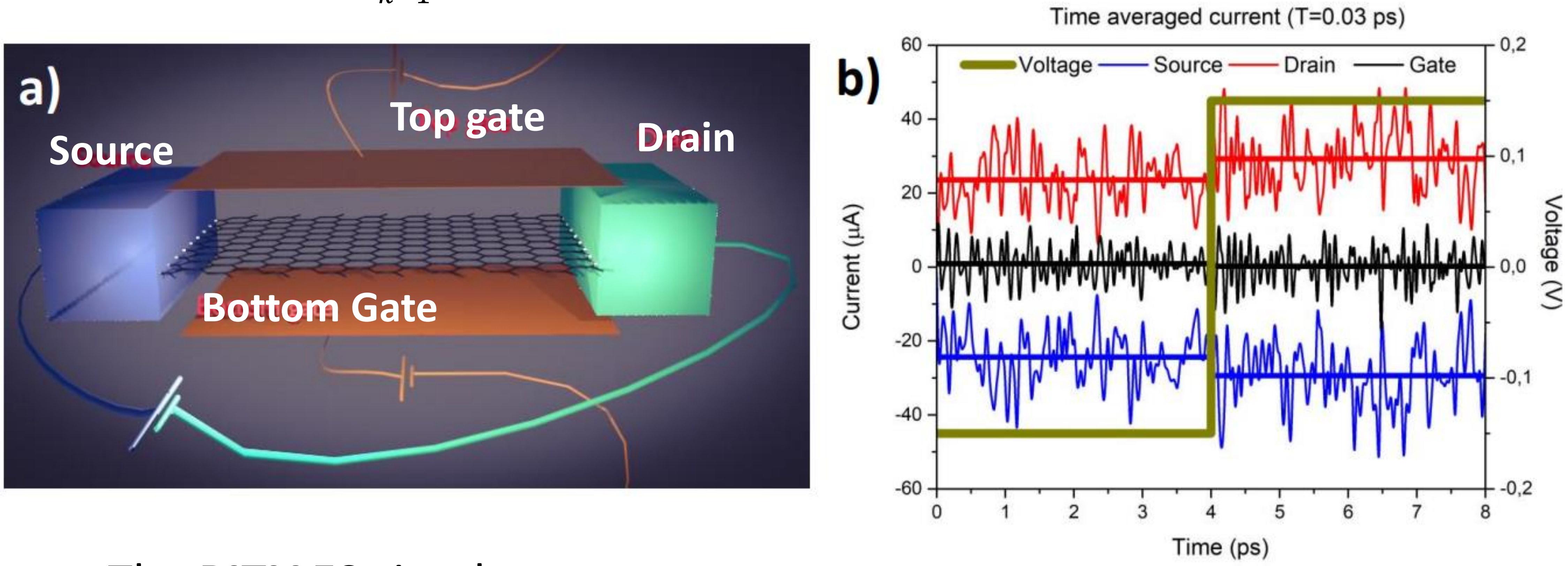

The **BITLLES** simulator.

D. Marian, N. Zanghì, and X. Oriols, Phys. Rev. Lett. 116, 110404 (2016)
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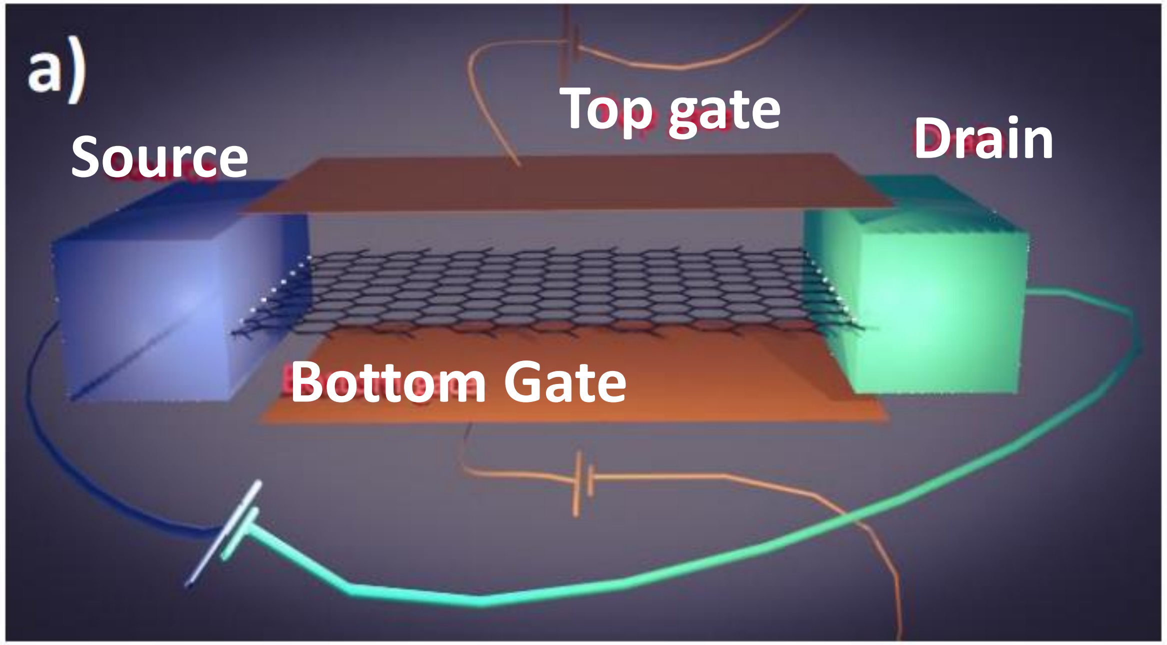

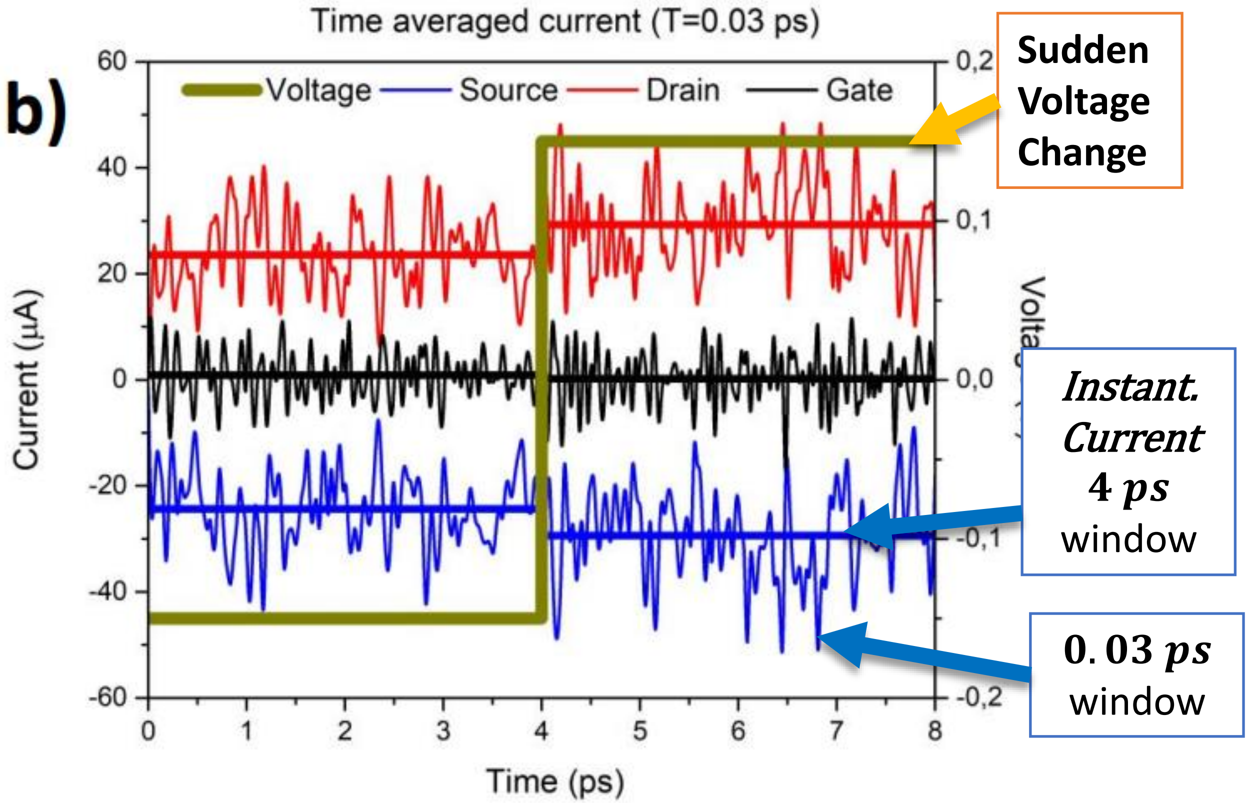

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Conclusions

- **Weak Values** (e.g. the ψ) **characterize quantum systems**
- They are **experimentally measurable** through **averages**
- **"Any" simulated result is** thus a **prediction** for an **experiment**
- They **promote Bohmian Mechanics to** a **practical tool :**
  - Resolve **pathological scenarios**
  - **Non-Markovian SSE** Toolbox

Thank you
for your attention!

Questions?

(d.1.) Quantum Work?

$$W(t_1, t_2) \sim E(t_1), E(t_2), path(t_1, t_2)$$

D. H. Kobe, J. Phys. A Math. Theor. 40, 5155 (2007)

R. Sampaio, S. Suomela, T. Ala-Nissila, J. Anders, and T. Philbin, Phys. Rev. A 97, 012131 (2018)

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$$W(t_1, t_2) \sim E(t_1), E(t_2), path(t_1, t_2)$$

- **Path *if* measurement** $\neq$ ***if not*** measurement
- **Energy undefined if not measurement**

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- **Path *if* measurement ≠ *if not*** measurement
- **Energy undefined if not measurement**

No uncontextual quantum **work?**

D. H. Kobe, J. Phys. A Math. Theor. 40, 5155 (2007)

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(d.1.) Quantum Work?

$$W(t_1, t_2) \sim E(t_1), E(t_2), path(t_1, t_2)$$

Well...

Energy of the system in $\vec{x}, t$

$$\mathcal{E}^{\psi}(\vec{x}, t) = Re\left\{ \frac{\langle \vec{x} | \hat{H}(t) | \psi(t) \rangle}{\langle \vec{x} | \psi(t) \rangle} \right\} = \frac{\vec{\wp}^{\,\psi}(\vec{x}, t)^2}{2m} + V(\vec{x}, t) + Q(\vec{x}, t)$$

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⬇

Work on the ξ-th trajectory

$$W^{\xi}(t_1, t_2) \coloneqq \int_{t_1}^{t_2} \frac{d}{dt} \mathcal{E}^{\psi}\left(\vec{x}^{\xi}(t), t\right) dt = \mathcal{E}^{\psi}\left(\vec{x}^{\xi}(t_2), t_2\right) - \mathcal{E}^{\psi}\left(\vec{x}^{\xi}(t_1), t_1\right)$$

D. H. Kobe, J. Phys. A Math. Theor. 40, 5155 (2007)

R. Sampaio, S. Suomela, T. Ala-Nissila, J. Anders, and T. Philbin, Phys. Rev. A 97, 012131 (2018)

II – Unprecedented Characterization

(d.1.) Quantum Work?

Well…

$$W(t_1, t_2) \sim E(t_1), E(t_2), path(t_1, t_2)$$

Energy of the system in $\vec{x}, t$

$$\mathcal{E}^{\psi}(\vec{x}, t) = Re\left\{ \frac{\langle \vec{x} | \widehat{H}(t) | \psi(t) \rangle}{\langle \vec{x} | \psi(t) \rangle} \right\} = \frac{\vec{\wp}^{\psi}(\vec{x}, t)^2}{2m} + V(\vec{x}, t) + Q(\vec{x}, t)$$

⬇

Work on the ξ-th trajectory

$$W^{\xi}(t_1, t_2) := \int_{t_1}^{t_2} \frac{d}{dt} \mathcal{E}^{\psi}\left(\vec{x}^{\xi}(t), t\right) dt = \mathcal{E}^{\psi}\left(\vec{x}^{\xi}(t_2), t_2\right) - \mathcal{E}^{\psi}\left(\vec{x}^{\xi}(t_1), t_1\right)$$

⬇

Work on the quantum **system**

$$\langle W(t_1, t_2) \rangle := \lim_{|\sigma| \to \infty} \frac{1}{|\sigma|} \sum_{\xi \in \sigma} W^{\xi}(t_1, t_2) = \langle \psi(t_2) | \widehat{H}(t_2) | \psi(t_2) \rangle - \langle \psi(t_1) | \widehat{H}(t_1) | \psi(t_1) \rangle$$

D. H. Kobe, J. Phys. A Math. Theor. 40, 5155 (2007)

R. Sampaio, S. Suomela, T. Ala-Nissila, J. Anders, and T. Philbin, Phys. Rev. A 97, 012131 (2018)

(d.2.) Two-time correlations

$$\left[\hat{G},\hat{J}\right] \neq \hat{0} \quad \Rightarrow \quad \left\langle \hat{G}(t_2)\hat{J}(t_1)\right\rangle = \langle\psi|\hat{G}(t_2)\hat{J}(t_1)|\psi\rangle \in \mathbb{C}$$

No well-defined two-time correlation?

D. Pandey, R. Sampaio, T. Ala-Nissila, G. Albareda, and X. Oriols, Phys. Rev. A 103, 052219 (2021).

(d.2.) Two-time correlations

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Well…

$$g^{\psi}(\vec{x},t) := Re\left\{\frac{\langle\vec{x}|\hat{G}|\psi(t)\rangle}{\langle\vec{x}|\psi(t)\rangle}\right\} \qquad j^{\psi}(\vec{x},t) := Re\left\{\frac{\langle\vec{x}|\hat{F}|\psi(t)\rangle}{\langle\vec{x}|\psi(t)\rangle}\right\}$$

$$\langle G(t_2)J\,(t_1)\rangle \coloneqq \lim_{|\sigma|\to\infty}\frac{1}{|\sigma|}\sum_{\xi\in\sigma} g^{\psi}\left(\vec{x}^{\xi}(t_1),t_1\right) j^{\psi}\left(\vec{x}^{\xi}(t_2),t_2\right)$$

D. Pandey, R. Sampaio, T. Ala-Nissila, G. Albareda, and X. Oriols, Phys. Rev. A 103, 052219 (2021).

III – A Recipe for non-Markovian Open Quantum Systems

$$\Psi(\vec{\boldsymbol{x}}_{\mathbf{1}}, \dots, \vec{\boldsymbol{x}}_{N}, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$$

$$\hat{\rho}_{1\dots N,E}(t)$$

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$$\Psi(\vec{\boldsymbol{x}}_{\mathbf{1}}, \ldots, \vec{\boldsymbol{x}}_{\boldsymbol{N}}, \vec{\boldsymbol{y}}_{\boldsymbol{Env}}, \boldsymbol{t})$$

$$\hat{\rho}_{1\ldots N,E}(t)$$

One non-M.
open QS

$$\Psi\left(\vec{\boldsymbol{x}}_{\mathbf{1}}, \ldots, \vec{\boldsymbol{x}}_{\boldsymbol{N}},;\ \vec{y}^{\xi}_{Env}(t), t\right)$$

$$\rightarrow\ \left|\psi^{\xi}(t); g_{\xi}(t)\right\rangle_{1\ldots N}$$

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$\rightarrow\ |\psi^{\xi}(t); g_{\xi}(t)\rangle_{1\ldots N}$

$\hat{\boldsymbol{\rho}}_{1\ldots N}(\boldsymbol{t})$

OK!

But… no $\mathcal{P}$ approx.!

III – A Recipe for non-Markovian Open Quantum Systems

$\Psi(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$

$\hat{\rho}_{1\dots N,E}(t)$

$$\Psi\left(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, ;\ \vec{y}^{\,\xi}_{Env}(t), t\right) \rightarrow \left|\psi^{\xi}(t); g_{\xi}(t)\right\rangle_{1\dots N}$$

One non-M. **open QS**

$\hat{\boldsymbol{\rho}}_{1\dots N}(\boldsymbol{t})$

OK!

But… no $\mathcal{P}$ **approx.!**

N non-M open QS!

$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\,\xi}_2(t), \dots, \vec{x}^{\,\xi}_N(t),\ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow \left|\psi^{\xi}_1(t); g_{\xi}(t)\right\rangle_1$$

…

$$\Psi\left(\vec{x}^{\,\xi}_1(t), \dots, \vec{x}^{\,\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N, \vec{y}^{\,\xi}_P(t), t\right) \rightarrow \left|\psi^{\xi}_N(t); g_{\xi}(t)\right\rangle_N$$

III – A Recipe for non-Markovian Open Quantum Systems

$\Psi(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$

$\hat{\rho}_{1\dots N,E}(t)$

One non-M. **open QS**

$$\Psi\left(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, ;\ \vec{y}^{\,\xi}_{Env}(t), t\right) \rightarrow \left|\psi^{\xi}(t); g_{\xi}(t)\right\rangle_{1\dots N} \Rightarrow \hat{\boldsymbol{\rho}}_{1\dots N}(\boldsymbol{t})$$

OK!

But… no $\mathcal{P}$ approx.!

N non-M open QS!

$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\,\xi}_2(t), \dots, \vec{x}^{\,\xi}_N(t),\ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow \left|\psi^{\xi}_1(t); g_{\xi}(t)\right\rangle_1 \Rightarrow \hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$$

…

$$\Psi\left(\vec{x}^{\,\xi}_1(t), \dots, \vec{x}^{\,\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N,\ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow \left|\psi^{\xi}_N(t); g_{\xi}(t)\right\rangle_N \Rightarrow \hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$$

III – A Recipe for non-Markovian Open Quantum Systems

$\Psi(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$

$\hat{\rho}_{1\dots N,E}(t)$

$\Psi\left(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, ; \ \vec{y}^{\,\xi}_{Env}(t), t\right) \rightarrow |\psi^{\xi}(t); g_{\xi}(t)\rangle_{1\dots N}$ ➡ $\hat{\rho}_{1\dots N}(\boldsymbol{t})$

One non-M. **open QS**

OK!
But… no $\mathcal{P}$ **approx.!**

N non-M open QS!

$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\,\xi}_2(t), \dots, \vec{x}^{\,\xi}_N(t), \ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_1(t); g_{\xi}(t)\rangle_1 \quad \Rightarrow \quad \hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$$

…

$$\Psi\left(\vec{x}^{\,\xi}_1(t), \dots, \vec{x}^{\,\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N, \vec{y}^{\,\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_N(t); g_{\xi}(t)\rangle_N \quad \Rightarrow \quad \hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$$

Available
$\mathcal{P}_k$ **apprx.!**

…

$\hat{\boldsymbol{\rho}}_{1\dots N}(t)$ **?**
Ψ **?**

III – A Recipe for non-Markovian Open Quantum Systems

$\Psi(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$

$\hat{\rho}_{1\dots N,E}(t)$

$$\Psi\left(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N,;\ \vec{y}^{\,\xi}_{Env}(t), t\right) \rightarrow \left|\psi^{\xi}(t); g_{\xi}(t)\right\rangle_{1\dots N}$$

One non-M. open QS

$\hat{\boldsymbol{\rho}}_{1\dots N}(\boldsymbol{t})$

OK!
But… no $\mathcal{P}$ approx.!

N non-M open QS!

$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\,\xi}_2(t), \dots, \vec{x}^{\,\xi}_N(t),\ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow \left|\psi^{\xi}_1(t); g_{\xi}(t)\right\rangle_1$$

$\hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$

…

$$\Psi\left(\vec{x}^{\,\xi}_1(t), \dots, \vec{x}^{\,\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N, \vec{y}^{\,\xi}_P(t), t\right) \rightarrow \left|\psi^{\xi}_N(t); g_{\xi}(t)\right\rangle_N$$

$\hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$

Available $\mathcal{P}_k$ apprx.!

$\hat{\boldsymbol{\rho}}_{1\dots N}(t)$ **?**

Ψ **?**

No need!

III – A Recipe for non-Markovian Open Quantum Systems

$\Psi(\vec{\boldsymbol{x}}_1,\ldots,\vec{\boldsymbol{x}}_N,\vec{\boldsymbol{y}}_{Env},\boldsymbol{t})$
$\hat{\rho}_{1\ldots N,E}(t)$

One non-M. **open QS**

$\Psi\left(\vec{\boldsymbol{x}}_1,\ldots,\vec{\boldsymbol{x}}_N,;\ \vec{y}^{\xi}_{Env}(t),t\right) \rightarrow \left|\psi^{\xi}(t);g_{\xi}(t)\right\rangle_{1\ldots N}$ → $\hat{\boldsymbol{\rho}}_{1\ldots N}(\boldsymbol{t})$

OK!
But... no $\mathcal{P}$ approx.!

N non-M open QS!

$$\Psi\left(\vec{\boldsymbol{x}}_1,\vec{x}^{\xi}_2(t),\ldots,\vec{x}^{\xi}_N(t),\ \vec{y}^{\xi}_P(t),t\right) \rightarrow \left|\psi^{\xi}_1(t);g_{\xi}(t)\right\rangle_1 \rightarrow \hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$$

...

$$\Psi\left(\vec{x}^{\xi}_1(t),\ldots,\vec{x}^{\xi}_{N-1}(t),\vec{\boldsymbol{x}}_N,\ \vec{y}^{\xi}_P(t),t\right) \rightarrow \left|\psi^{\xi}_N(t);g_{\xi}(t)\right\rangle_N \rightarrow \hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$$

Available $\mathcal{P}_k$ apprx.!

$\hat{\boldsymbol{\rho}}_{1\ldots N}(t)$ **?**
Ψ **?**
No need!

$$j^{\psi}_k(\vec{x},t) = Re\left\{\frac{\langle\vec{\boldsymbol{x}}_1,\ldots,\vec{\boldsymbol{x}}_N|\widehat{Id}_1\cdots\hat{J}_k\cdots\widehat{Id}_N|\psi\rangle}{\langle\vec{\boldsymbol{x}}_1,\ldots,\vec{\boldsymbol{x}}_N|\psi\rangle}\right\}$$

$$\mathcal{J}^{\psi}_T = \Sigma^N_{j=1}\, j^{\psi}_k$$

III – A Recipe for non-Markovian Open Quantum Systems

$$\Psi(\vec{\boldsymbol{x}}_1, \ldots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$$
$$\hat{\rho}_{1\ldots N,E}(t)$$

One non-M. **open QS**

$$\Psi\left(\vec{\boldsymbol{x}}_1, \ldots, \vec{\boldsymbol{x}}_N, ;\ \vec{y}^{\,\xi}_{Env}(t), t\right) \rightarrow |\psi^{\xi}(t); g_{\xi}(t)\rangle_{1\ldots N} \Rightarrow \hat{\boldsymbol{\rho}}_{1\ldots N}(\boldsymbol{t})$$

OK!
But... no $\mathcal{P}$ approx.!

N non-M open QS!

$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\,\xi}_2(t), \ldots, \vec{x}^{\,\xi}_N(t),\ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_1(t); g_{\xi}(t)\rangle_1 \Rightarrow \hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$$

...

$$\Psi\left(\vec{x}^{\,\xi}_1(t), \ldots, \vec{x}^{\,\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N,\ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_N(t); g_{\xi}(t)\rangle_N \Rightarrow \hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$$

Available $\mathcal{P}_k$ apprx.!

$\hat{\boldsymbol{\rho}}_{1\ldots N}(t)$?
Ψ ?

No need!

$$j^{\psi}_k(\vec{x}, t) = Re\left\{\frac{\langle \vec{\boldsymbol{x}}_1, \ldots, \vec{\boldsymbol{x}}_N | \widehat{Id}_1 \cdots \hat{J}_k \cdots \widehat{Id}_N | \psi\rangle}{\langle \vec{\boldsymbol{x}}_1, \ldots, \vec{\boldsymbol{x}}_N | \psi\rangle}\right\}$$

$$\mathcal{J}^{\psi}_T = \Sigma^{N}_{j=1}\, j^{\psi}_k$$

$$\mathbb{E}[\mathcal{J}\_T](t) = \lim_{|\sigma|\to\infty} \frac{1}{|\sigma|} \sum_{\xi\in\sigma} \mathcal{J}^{\psi}_T\left(\vec{x}^{\,\xi}_1(t), \ldots, \vec{x}^{\,\xi}_N(t), t\right)$$
$$= \langle \psi | \hat{\mathcal{J}} | \psi \rangle$$