











"Unveiling quantum phase transition by disorder and defects in 2D-materials: Jacutingaite family"

@ Jacutingaite-family : A class of topological materials

@ Topological insulating phase arisi Jacutingaite- family ordered and ranalloys.

@ Vacancies – driven Quantum Spin Hall on transition metal dichalcogenides.



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Topological Insulators are called "topological" because the wave functions describing their electronic states span a Hilbert space that has a nontrivial topology... they opened a new window for understanding the elaborate workings of nature.

> **Consequence?...** a gapless interface state necessarily shows up when the insulator is physically terminated and faces an ordinary insulator (including the vacuum).

Which invariant? (Time Reversal invariant)

 $\mathbb{Z}_{2} = \frac{1}{2\pi} \left[\oint_{\partial \operatorname{HBZ}} \mathrm{d}\boldsymbol{k} \cdot \boldsymbol{A}(\boldsymbol{k}) - \int_{\operatorname{HBZ}} \mathrm{d}^{2}k \,\Omega_{z}(\boldsymbol{k}) \right] \operatorname{mod}(2)$

 $\vec{A}_n(\vec{k}) = -i \langle u_{n\vec{k}} | \vec{\nabla}_k | u_{n\vec{k}} \rangle \qquad \vec{\Omega}_n(\vec{k}) = \vec{\nabla}_k \times A_n(\vec{k})$

Isolantes topológicos Z₂ = 1 Isolantes triviais Z₂ =0

Berry connection

Berry curvature



Why Jacutingaite-family ?

New candidates for topological materials: (i) large topological gap. (ii) structural stability. (iii) accessible synthesis route.



@ Pt₂HgSe₃ is a naturally occurring mineral discovered in Brazil in 2008.

@ In 3D-form is predict a dual topology , weak topological insulator and topological crystaline insulator.

@ Its monolayer form, a place for manifestation of the Kane-Mele topological phase

@ Also Pt₂HgSe₃ has been synthesized and also Pdbased in the same structure Pd₂HgSe₃.



companies in the world -Vale do Rio Doce

Platinum enrichment at low temperatures and related microstructures, with examples of hongshiite (PtCu) and empirical 'Pt₂HgSe₃' from Itabira, Minas Gerais, Brazil 2008

A. R. Cabral,¹ H. F. Galbiatti,² R. Kwitko-Ribeiro³ and B. Lehmann⁴

¹Department of Geology: Exploration Geology, Rhodes University, PO Box 94, Grahamstown 6140, South Africa; ²Companhia Vale do Rio Doce, 35900-900 Itabira–MG, Brazil; ³Desenvolvimento de Projetos Minerais, Companhia Vale do Rio Doce, Rodovia BR 381/km 450, 33040-900 Santa Luzia–MG, Brazil; ⁴Institut für Mineralogie und Mineralische Rohstoffe, Technische Universität Clausthal, Adolph-Roemer-Str. 2A, Clausthal-Zellerfeld, D-38678, Germany @ Pt₂HgSe₃ is a naturally occurring mineral discovered in Brazil in 2008.

@ In 3D-form is predict a dual topology, weak topological insulator and topological crystaline insulator.

@ its monolayer form, a place for manifestation of the Kane-Mele topological phase

@ Also Pt₂HgSe₃ has been synthesized and also Pdbased in the same structure Pd₂HgSe₃.

First theoretical paper

Prediction of a Large-Gap and Switchable Kane-Mele Quantum Spin Hall Insulator

Antimo Marrazzo,^{*} Marco Gibertini, Davide Campi, Nicolas Mounet, and Nicola Marzari[†] Theory and Simulation of Materials (THEOS) and National Centre for Computational Design and Discovery of Novel Materials (MARVEL), École Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland



its monolayer(2D) form, a place for manifestation of the Kane-Mele topological phase. Whereas in 3D-form is predict a dual topology, weak topological insulator and topological crystaline insulator.

PRL 2018

Its atomic structure can be viewed as a the transition metal dichalcogenide (TMD) PtSe2 with a structural phase where ¹/₄ of the chalcogenides are replaced by Hg.





Letter



2024

npj Quantum Materials

2021

...

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ARTICLE OPEN

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Pressure-induced superconductivity and structure phase transition in Pt_2HgSe_3

Cuiying Pei@^{1,8}, Suhua Jin^{1,8}, Peihao Huang^{2,8}, Anna Vymazalova³, Lingling Gao[®], Yi Zhao[®], Weizheng Cao[®], Changhua Li¹, Peter Nemes-Incze[®], Yulin Chen^{®1,5,6}, Hanyu Liu^{®2,7,≅}, Gang Li^{®1,5,≅} and Yanpeng Qi^{®1,≅}

PHYSICAL REVIEW LETTERS 124, 106402 (2020)

Bulk and Surface Electronic Structure of the Dual-Topology Semimetal Pt2HgSe3

I. Cucchi[®],¹ A. Marrazzo[®],² E. Cappelli,¹ S. Riccò,¹ F. Y. Bruno[®],^{1,3} S. Lisi,¹ M. Hoesch[®],^{4,5} T. K. Kim[®],⁴ C. Cacho,⁴ C. Besnard,¹ E. Giannini[®],¹ N. Marzari[®],² M. Gibertini[®],^{1,2} F. Baumberger[®],^{1,6} and A. Tamai[®],^{1,*}



pubs.acs.org/NanoLett

Signature of Large-Gap Quantum Spin Hall State in the Layered Mineral Jacutingaite

Konrád Kandrai, Péter Vancsó, Gergő Kukucska, János Koltai, György Baranka, Ákos Hoffmann, Áron Pekker. Katalin Kamarás. Zsolt E. Horváth. Anna Vvmazalová. Levente Tapasztó.



Pt₂HgSe₃ Jacuntingaite-family >>> M₂NX₃



18

Termodinamic and mechanical stables

27 materials

M= Ni, Pd, Pt; N= Zn, Cd, Hg; X= S, Se, Te

Periodic Table of the Elements

IA																	VIIIA
1 H dragen 1000 1	2 IIA				Atomic Number	- Hydr	ogen +	Symbol				13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	He
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87 Fr	Ra	89-103 Actinides	104 Rf Rutharterdium QNT 2480-2010-2	105 Db Dubnium GMB	Seatoryum Califi Laborood	107 Bh Bohrium 020	108 Hs Rassium (277) 248-212-242	109 Mt Molterium (270)	110 Darmstadtium 2409 Daniel	III Rg Reenternium GRD 2400 DD22	112 Copernicium (240) 348-5025-82	Nitesium Cito Sale Differ	Fierovium GHD 2410 DODDes	Mc Mc Moscowiam GNU Mascowiam	Lv Lv Divermentium (212) 2450-220-04	TS TS Tecnessive CNO DATE DATE	
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		Actinium (220) 24-8-20-8-4-2	Therium 20204 24802/882	Protactinium 20164 24/8-32-35-52	Uranium 238.03 24-16-02-6-2	Neptunium GID 14-B-32-D1-7	Putersium GAO 36.5.22.342	Americium GLD 2476207547	Cm Gati 24/5/2/24/2	Bk Berkelium (MT) 24-10-22-4-2	Catifornium Catifornium DISP 24/8/202842	Es Einsteinium CHD 24/632/2442	Fm	Md Mendelevium GME 248:25:3442	No Notation (19) 248-202042	Lawrencium GMD 26/6/201043	

We employ density functional theory simulations (DFT)



Jacutingaite-like share the same backbone geometry of TMD(MX₂)-1T >--> M₂NX₃



 $PtSe_2 > ---- > Pt_2HgSe_3$ (1/4) Se \rightarrow Hg



The chalcogenide atoms (X) are partially replaced by transition metals (N), $M X_2 \rightarrow M_2 N X_3$, resulting in buckled N -M-N bonds. M₂NX₃



The lattice parameter of M_2NX_3 , are pratically independent of transition metal

The *N* atoms form triangular lattices on the opposite sides of the MX_2 host, which in turn are rotated by 60° with respect to each other, giving rise to a buckled hexagonal lattice.

* And the equilibrium lattice constant of Pt₂NSe₃ for N=Zn,Cd,Hg differ by less than 9% compared with 1T PtSe2



Pd (g) 0.8 • Zn 0.6 0.1 0 2 0.4 Cd ▲ Hgリ 0.2 20200000000

Cleavage energy($(M_2NX_3 bulk)$ - the dashed line indicating the graphite cleavage barrier.

Schematic band structure around Fermi-energy M< >K Direction



 M_2NX_3











Gap in Dirac-point (K)

(27 materials)

No-SOC X SOC





Replacing transition metals and chalcogen



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Article Tilkerodeite, Pd₂HgSe₃, a New Platinum-Group Mineral from Tilkerode, Harz Mountains, Germany

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- * Correspondence: chima@caltech.edu



• 2020





Then ... Pt₂HgSe₃-family

a) 12 Topological Insulators(Kane-Mele), 8 Topological Semi-Metal and 07 Trivial

b) The cleavage energies showed that these systems can be isolated in monolayers by exfoliation methods.

c) Pt_2ZnSe_3 and Pt_2ZnTe_3 , present larger values of topological band gap compared with that of jacutingaite (Pt_2HgSe_3).

....

"Unveiling quantum phase transition by disorder and defects in 2D-materials: Jacutingaite family"



@ Jacutingaite-family : A class of topological materials

@ Topological insulating phase arising in Jacutingaite- like ordered and random alloys.

@ Vacancies –driven Quantum Spin Hall on transition metal dichalcogenides.



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Topological insulating phase arising in transition metal dichalcogenide alloy

PtSe₂ is a semiconductor with trivial bandgap
>> 25% of Se substituted by Hg >>->>>
Pt₂HgSe₃ is a topological insulator with a large bandgap



Ordered alloys



Pt(Hg_xSe_{1-x})₂ with x=0.25, 0.50, 0.75 and 1.0



Pt GRAY Hg PURPLE Se GREEN

$Pt(Hg_xSe_{1-x})_2$ with x=0.25, 0.50, 0.75 and 1.0



Metallic

Topological phase arise due to hybridization of the Hg s-orbitals with the Pt-dorbitals

Insulator

 $Pt(Hg_xSe_{1-x})_2$ with x=0.25, 0.50, 0.75

All topological materials







Hg–Hg-interaction embedded in PtSe₂ matrix

$$E_s = \left(E_{\text{system}} + n\,\mu_{\text{Se}} - E_{\text{PtSe}_2} - n\,\mu_{\text{Hg}}\right)/n$$

Tetragonal supercell 18.7A x 32.4A

8x8 with 192 atoms.



>Substitutional energy for a single Hg (n=1)

$$E_s = \left(E_{\text{system}} + n\,\mu_{\text{Se}} - E_{\text{PtSe}_2} - n\,\mu_{\text{Hg}}\right)/n$$





>Substitutional energy for interacting Hg pairs (n=2)

$$E_{s} = (E_{system} + n \mu_{Se} - E_{PtSe_{2}} - n \mu_{Hg}) / n$$

$$\underbrace{5.4}_{(k)} \underbrace{5.2}_{(k)} \underbrace$$





















For distances greater than 11A NO change in energy

Tetragonal supercell 18.7A x 32.4A





Pair most stable Hg⁰—Hg³ bonds sharing the same Pt atoms

Hg-Pt-Hg exactly that found(that we expected) in the Pt₂HgSe₃ crystal





• The Hg "wavefunctions" are overlaping with each other up to ~1,1nm, defining a localization length







$Pt(Hg_xSe_{1-x})_2 >> Hg_{Se}$ preserving the host 1T

*To provide a more realistic description of random alloys > SQS (special quasi random) approach

from x=0.25 to x=0.75 (Zunger et al PRL 1990)





The blue point represents the eigenvalue of the Hamiltonian. The blue line is just a guide to the eye

For a non-periodicity of the random alloy, we have computed a real space invariant : Spin Bott-index, which is equivalent to the spin-Chern number.





For a non-periodicity of the random alloy, we have computed a real space invariant :Spin Bott-index, which is equivalent to the spin-Chern number.



The blue point represents the eigenvalue of the Hamiltonian. The blue line is just a guide to the eye

For a non-periodicity of the random alloy, we have computed a real space invariant : Spin Bott-index, which is equivalent to the spin-Chern number.



Lowering Hg concentration, $\neq \dots > 0$, will lead the Pt(Hg_xSe_{1-x})₂ system to trivial phase PtSe₂.

Realistic disordered systems, within the SQS approach for lower concentrations increases the computational cost(cell size >10³

atoms.)

D > 11.24

• The Hg wavefunctions are overlaping with each other up to ~1,0nm, defining a localization length



Topological limit

* x---> 0 will lead the Pt(Hg_xSe_{1-x})₂ ---> PtSe₂ trivial phase

** We will look at a NON-Trivial ←> Trivial based on electron percolation limit

 $c^{(s)} = c_{+} - c_{-}$, (Chern number, integration over a closed surface of Berry connection) $c_{j} = \sum_{n}^{\text{occup}} \oint_{C} \vec{A}_{n}^{(j)} \cdot d\vec{k}, \quad \vec{A}_{n}^{(\sigma)} = i\langle n, \vec{k}, \sigma | \nabla_{k} | n, \vec{k}, \sigma \rangle$

>fully localized wave functions are eigenfunction of the position operator:

$$\hat{x} = i\nabla_k$$

$$\therefore c_{\sigma} = \sum_{n}^{\text{occup}} x_n \oint_C dk = 0$$

Non-zero Chern number requires the electronic states to **NOT** be completely localized (necessary but not sufficient).... For QSH phase.





>Mathematically it can be found a threshold of rigid disk concentration where a random distribution guarantee a percolation in hexagonal lattices.

$$\phi_c = 1 - e^{-n_c} = 0.69$$

Using the disks-diameters 11A aWe stimated limit concentration..

P.Suding and R.M.Ziff Phys.Rev. 60, 275 (99)

















>Mathematically it can be found a threshold of Hg concentration where a random distribution guarantee a percolation of the electrons hopping the Hg wave functions (i.e. a non localization limit)

$$\phi_c = 1 - e^{-n_c} = 0.69$$

$$n_c = A_I / A_{UC}$$

>This sets the necessary topological limit for random alloy at x ~ 10%



Linear conductance Landauer-Buttiker formula





Pt₂HgSe₃/Hg_{Se}



Scattering Region



Lead



Scattering Region

Random Substitutional Defect - Stoichiometry break











- By Increasing the ribbon width, one can recovers some of the transport channels close to Fermi level
- Less influence from the edge states

Conclusion

*Based on ab initio calculations, we have studied the energetic stability and trivial > non-trivial topological phase transition in single-layer $PtSe_2$ mediated by substitutional Hg_{Se} atoms, $Pt(Hg_xSe_{1-x})_2$ alloys.

** We found an energetic preference for a random distribution of Hg with x= 25% (jacutingaite's stochiometry), with respect to the PtSe₂ host ruled by the configurational entropy, for KT> 15meV and Random alloys with x=50%, PtHgSe , become more stable for KT>32meV

*** The QSH phase against the random distribution of Hg_{se} substitutional atoms has been verified for alloy concentration between 25% and 75%

****. With percolation model we estimated a threshold concentration of about 9% for topological non-trivial>trivial transition in $Pt(Hg_xSe_{1-x})_2$ random alloys



Vacancies centers is the simplest defect but can provide drive exotic effects as charge density waves in indium (In) nanowires,ferromagnetism on trasition metal dichalcogenides(TMD),transitions Metal-Insulators in GeSbTe(IV-VI), negative U in Si....etc





We investigate the trivial 2D semiconductor PtSe2 Z2=0



pubs.acs.org/NanoLett

Atomic-Level Dynamics of Point Vacancies and the Induced Stretched Defects in 2D Monolayer PtSe₂

Jun Chen,^{\perp} Jiang Zhou,^{\perp} Wenshuo Xu, Yi Wen, Yuanyue Liu,^{*} and Jamie H. Warner^{*}





Se vacancy formation on PtSe₂ can controlled by electron radiation...recent advances on controlled Atomic positioning on surfaces by AFM/STM TIPS Chalcogens vacancies: universal schematic of band structure of each semiconducting phase, respectively, with and without spin-orbit

each semiconducting phase, respectively, with and without spin-orbit corrections.



transition metal dichalcogenides.



i Marana



Se vacancy introduces localized states in the host PtSe2 bandgap

Three lone pairs arise in the Pt atoms neighboring the vacancy.

Increasing the vacancy density, those states can interact forming energy bands.

Orbitas with major contribuition from Pt dxz/dyz with its interactions mediated by the host Se p orbitals

PtSe₂ is a trivial semiconductor with an energy gap of **1.2** eV.











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Amorphous Bi₂Se₃ structural, electronic, and topological nature from first principles

Bruno Focassio^{1,2,*} Gabriel R. Schleder^{1,2,3} Felipe Crasto de Lima^{2,4} Caio Lewenkopf^{5,5} and Adalberto Fazzio^{2,1,4,†} ¹Center for Natural and Human Sciences, Federal University of ABC (UFABC), 09210-580 Santo André, São Paulo, Brazil ²Brazilian Nanotechnology National Laboratory (LNNano), CNPEM, 13083-970 Campinas, São Paulo, Brazil

spin Bott index is defined as

$$P_{\pm} = \sum_{n}^{N_{\rm occ}/2} |\phi_n^{\pm}\rangle \langle \phi_n^{\pm}|, \qquad (5)$$

$$B_s = \frac{1}{2}(B_+ - B_-),$$

where $|\phi_n^{\pm}\rangle$ are the eigenstates of P_z with eigenvalues S_{\pm} . In the case of 2D systems, for each spin sector one then constructs the projected position operators

$$U_{\pm} = P_{\pm} e^{i2\pi X} P_{\pm} + (I - P_{\pm}), \tag{6}$$

$$V_{\pm} = P_{\pm} e^{i2\pi Y} P_{\pm} + (I - P_{\pm}), \tag{7}$$

where X and Y are diagonal matrices with the x and y components of the spatial coordinate of each orbital site rescaled

$$B_{\pm} = \frac{1}{2\pi} \operatorname{Im}\{\operatorname{Tr}[\log(V_{\pm}U_{\pm}V_{\pm}^{\dagger}U_{\pm}^{\dagger})]\}.$$

For a non-periodicity of the random alloy, we have computed a real space invariant : Spin Bott-index, which is equivalent to the spin-Chern number.

been explained in the literature [18,19,24,30,31]. First, one constructs the projector operator of the occupied states below

Given the method of calculating the Bott index, now we give a general construction of the spin Bott index. One begins by introducing a projected spin operator

$$P_z = P\hat{s}_z P, \tag{5}$$

where $\hat{s}_z = \frac{\hbar}{2}\sigma_z$ is the spin operator (σ_z is the Pauli matrix). For a spin-conserving model, \hat{s}_z commutes with the Hamiltonian H and P_z , the Hamiltonian as well as eigenvectors can be divided into spin-up and spin-down sectors. Thus, the eigenvalues of P_z consist of just two nonzero values $\pm \frac{\hbar}{2}$. For systems without spin conservation (for example, the Kane-Mele model with nonzero Rashba terms which will be discussed later), the \hat{s}_z and H no longer commute. The spectrum of P_z spreads toward zero. However, as long as the spin-mixing term is not too strong, the eigenvalues of P_z remain two isolated groups which are separated by zero. Since the rank of P_z is N_{occ} , the number of positive eigenvalues equals to the number of negative eigenvalues, which is one half of N_{occ} . The corresponding eigenvalue problem can be denoted as

$$P_{z}|\pm\phi_{i}\rangle = S_{\pm}|\pm\phi_{i}\rangle. \tag{6}$$

In this way, one can construct new projector operators

$$P_{\pm} = \sum_{i}^{N_{\text{occ}}/2} |\pm \phi_i\rangle \langle \pm \phi_i|, \qquad (7)$$

which satisfy $P = P_+ \oplus P_-$, and projected position operators

$$U_{\pm} = P_{\pm} e^{i2\pi X} P_{\pm} + (I - P_{\pm}),$$
(8)
$$V_{\pm} = P_{\pm} e^{i2\pi Y} P_{\pm} + (I - P_{\pm}),$$
(9)

The Bott indices for two spin sectors are now given by [27–29,32]

$$B_{\pm} = \frac{1}{2\pi} \operatorname{Im} \{ \operatorname{tr}[\log(\tilde{V}_{\pm} \tilde{U}_{\pm}^{\dagger} \tilde{U}_{\pm}^{\dagger})] \}.$$
(10)

Finally, we define the spin Bott index as the half-difference between the Bott indices for the two spin sectors

$$B_s = \frac{1}{2}(B_+ - B_-). \tag{11}$$

Similar to the spin Chern number [8–10,25,26] the spin Bott index is a well-defined topological invariant. The spin Bott index is also directly related to the \mathbb{Z}_2 topological invariant. Its robustness is due to the existence of two spectral gaps: the insulating gap of the Hamiltonian and the spectral gap of the projected spin operator P_z . As long as the two gaps persist, the computational formalism of the spin Bott index can be applied. The spin Bott index is applicable to quasiperiodic and nonperiodic systems, which provides especially a useful tool to determine the electronic topology of those systems without periodicity.

FIG. 1. (a) Topological phase transition in the Haldane model. The parameters are t = -1 and $t_2 = 0.15e^{-i\pi/3}$. (b) Topological phase transition in the Kane-Mele model. The parameters are t = 1, $\lambda_{SO} = 0.3$, and $\lambda_R = 0.25$. The Bott *B* (spin Bott B_s) index is consistent with the Chern number (\mathbb{Z}_2 invariant) except around the phase transition point. This is because we use a relatively small supercell to calculate the Bott index. The small divergence would disappear if a larger supercell is used in the calculation of (spin) Bott index. The calculated (spin) Bott index with SVD shows a better performance than the one without SVD.

Huang H and Liu F 2018 Theory of spin Bott index for quantum spin Hall states in nonperiodic systems *Phys. Rev.* B 98 125130







>The Hg wavefunctions are interacting with each other up to ~1nm, defining a localization length



$$c^{(s)} = c_+ - c_-$$

$$c_{\sigma} = \sum_{n}^{\text{occup}} \oint_{C} \vec{A}_{n}^{(\sigma)} \cdot d\vec{k}$$
$$\vec{A}_{n}^{(\sigma)} \cdot \vec{k}$$

$$\hat{A}_{n}^{(\sigma)} = i\langle n, k, \sigma | \nabla_{k} | n, k, \sigma \rangle$$

>fully localized wave functions are eigenfunction of the position operator: $\hat{x} = i \nabla_k$

$$\therefore c_{\sigma} = \sum_{n}^{\text{occup}} x_n \oint_C dk = 0$$

