# What is Current?

**Robert Eisenberg**
**Illinois Institute of Technology**
**Rush University**
**University of Illinois Chicago**

1

# Thanks to Xavier Oriols

## for inviting me to speak at



Even though I speak more about ions than electrons!

# Thanks to Xavier Oriols for inviting me to speak at

# Special Session in Honor of Dave Ferry

*I will try to convince you:*

# Kirchhoff's law should use TOTAL current
## that includes displacement current $\varepsilon_0 \partial E / \partial t$
### It should not use the flow of electrons

## Kirchhoff's Law says
## "What flows in, flows out,
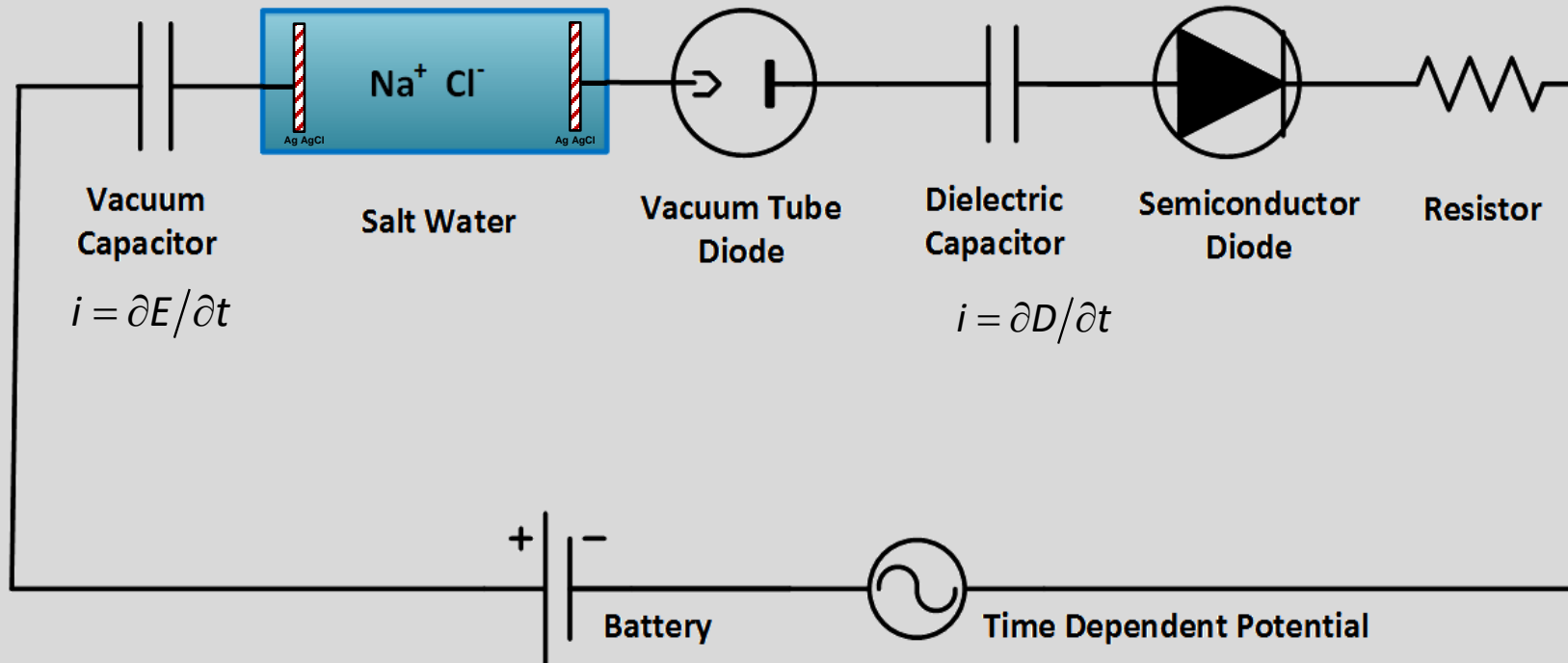## With<u>out</u> accumulation."

## but, according to Maxwell,
## electrons accumulate!

# Circuits are Special:
## TRIVIAL for engineers and PROFOUND for everyone else

**Total Current is the Same in Series Systems**
**Independent of Mechanism of Charge Movement**
*Kirchhoff Coupling*

Vacuum Capacitor

Salt Water — Na$^+$ Cl$^-$ — Ag AgCl   Ag AgCl

Vacuum Tube Diode

Dielectric Capacitor

Semiconductor Diode

Resistor

$i = \partial E / \partial t$

$i = \partial D / \partial t$

+ − Battery

Time Dependent Potential

**What is Current?**

# Current is Defined in Physics as that which makes a magnetic field

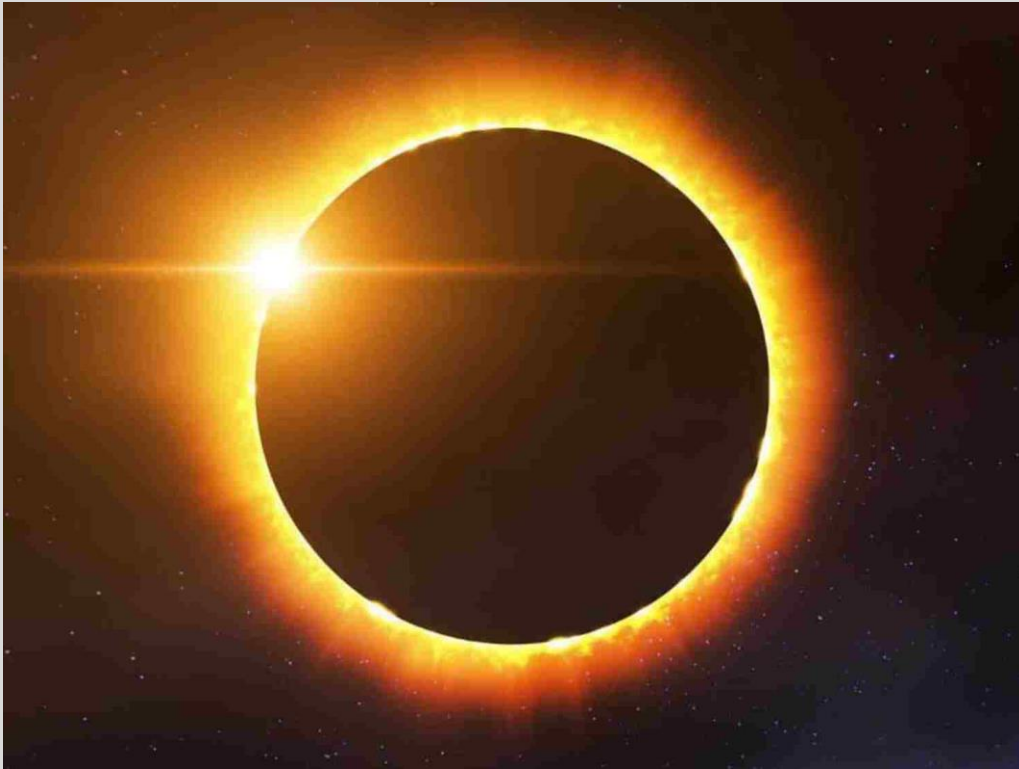# Current is NOT the Flow of Charge

## How do we know that?

# Magnetic Fields Exist in Vacuum

**Charge and Flow of Charge are ZERO in a vacuum**

## Magnetic Fields in Vacuum Create Electromagnetic Waves
### *LIGHT*

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

$c = 1/\sqrt{\varepsilon_0 \mu_0}$ = **velocity of light**

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

# Light travels through the Vacuum of Space
## ethereal current $\varepsilon_0 \partial \mathrm{E}/\partial t$ flows in vacuum of space,
### once thought to be filled with an 'aether'

Jeans 1908. The mathematical theory of electricity and magnetism.
Whittaker 1951. A History of the Theories of Aether & Electricity.
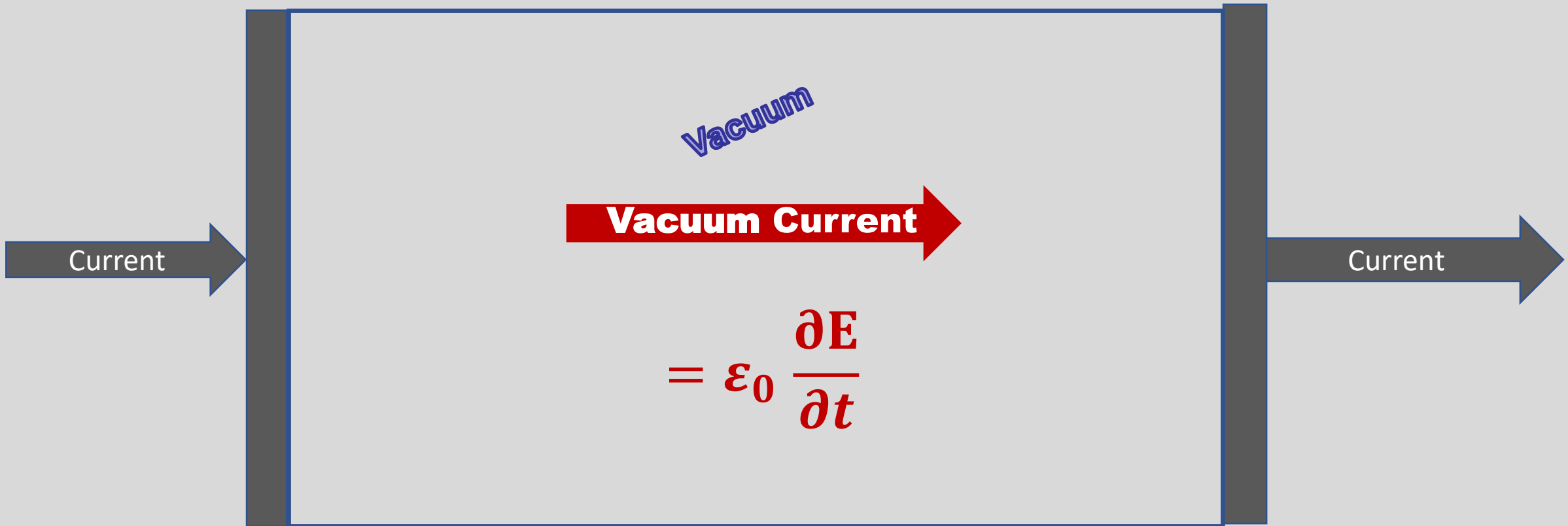Simpson 1998. Maxwell on the Electromagnetic Field: A Guided Study.



$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

**Wave Equation**
Corollary of
Maxwell Equations

$c = 1/\sqrt{\varepsilon_0 \mu_0}$ = velocity of light

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

# Well known Example
### taught,
### *or should be taught,*
### In First Year of Physics

**Vacuum**

**Current** **Vacuum Current** **Current**

$$= \varepsilon_0 \frac{\partial \mathrm{E}}{\partial t}$$

**Vacuum current = Ethereal current = Displacement Current**
**All are names for the same thing $\varepsilon_0\, \partial \mathrm{E}/\partial t$**

# Maxwell Ampere Law in a Vacuum

**Magnetism**

$\downarrow$

$$\frac{1}{\mu_0} \, \textbf{curl} \, \textbf{B} = \varepsilon_0 \, \underbrace{\frac{\partial \textbf{E}}{\partial t}}$$

**CURRENT**

*Vacuum contains no charge
and thus flow of charge* $\textbf{J} = \textbf{0}$

**No known error between stars, inside atoms
on all time scales, even those of thermal motion**

# Maxwell Ampere Law in Matter

*No known error, inside atoms or between stars*

**Magnetism**
$\downarrow$

**Universal Displacement Current**

$$\frac{1}{\mu_0} \, \mathbf{curl} \, \mathbf{B} = \mathbf{J} + \boldsymbol{\varepsilon}_0 \, \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}_{total}$$

*Fundamental Experimental Result*
*in Physics*
$\varepsilon_0 \partial \mathbf{E}/\partial t$ **exists everywhere,** *e.g.,*
*inside atoms*

**J is the flow of all charge,**
*however brief, small or transient.*
**J includes the polarization current**
**of idealized dielectrics** $(\varepsilon_r - 1)\varepsilon_0 \, \partial E/\partial t$

# Maxwell-Ampere Equation

$$\frac{1}{\mu_0} \, \mathbf{curl} \, \mathbf{B} = \mathbf{J}_{total} \qquad \qquad \textbf{(1)}$$

$\mathbf{J}_{total}$ includes the flux of charge with mass, however brief, small, or transient.
$\mathbf{J}_{total}$ the polarization current of idealized dielectrics $(\varepsilon_r - 1)\varepsilon_0 \, \partial E / \partial t$

# Identity

$$\mathbf{div} \, \mathbf{curl} \, anything = \mathbf{0} \qquad \qquad \textbf{(2)}$$

## Equations imply, by mathematics alone,

### Kirchhoff Current Law for Fields

$$\mathbf{div} \, \mathbf{J}_{total} = \mathbf{div} \, \mathbf{curl} \, \mathbf{B} = \mathbf{0}$$

**Kirchhoff Current Law for Fields**

$$\mathbf{div\ J}_{total} = \mathbf{div\ curl\ B} = 0$$

## Maxwell-Ampere Equation
*implies*
## Zero Accumulation of Total Current

$\mathbf{J}_{total}$ includes the flux of charge with mass, however brief, small, or transient.
$\mathbf{J}_{total}$ includes the polarization current of idealized dielectrics $(\varepsilon_r - 1)\varepsilon_0\, \partial E / \partial t$

# Maxwell $\quad$ div $\mathbf{J}_{total} = 0$

## and

# Classical Kirchhoff $\quad$ div $\mathbf{J} = 0$

# DISAGREE

$$\mathbf{J}_{total} = \mathbf{J} + \varepsilon_0 \, \partial \mathbf{E}/\partial t$$

**Usual derivation of Circuit Kirchhoff Discusses only flux J of charges**
**Derivation should discuss $\mathbf{J}_{total}$**

**Kirchhoff's Law says
"What flows in, flows out,
Without accumulation."**

**but, according to Maxwell,
electrons accumulate !**

*Conclusion*
**Kirchhoff's law should use TOTAL current**

**Paradigm Change**

# The flux of charges accumulates.

as 'free' charge $\rho$

by Continuity Equation $\mathbf{div\,J} = -\varepsilon_0 \; \partial\rho/\partial t \quad \Rightarrow \quad \mathbf{div\,J} \neq \mathbf{0}$

$\mathbf{J}$ does accumulate; $\quad \mathbf{J}_{total}$ does not accumulate,

If $\mathbf{div}\,(\mathbf{J} + \boldsymbol{\varepsilon_0}\; \partial\mathbf{E}/\partial t) = \mathbf{0}, \quad \mathbf{div\,J} \neq \mathbf{0}$

**Kirchhoff's Law should
Be about TOTAL current**

# Important Applications
## in Engineering

**like computer programs *Spice* and *Circuit Board Design***

# should use TOTAL current
**because use of flux is Actually Inconsistent with Maxwell equations**

# Numerical treatments of Electrodynamics
# must include the Displacement Current $\varepsilon_0\,\partial\mathbf{E}/\partial t$

**Numerical Treatments with dielectric term $\varepsilon_r\varepsilon_0\,\partial\mathbf{E}/\partial t$ automatically include Displacement Current because $\varepsilon_r \geq 1$**

# Optimal Numerical Treatment
# of Total Current
# is put forward in the following references

**Ji, Lijie, Pei Liu, Zhenli Xu, and Shenggao Zhou. "Asymptotic Analysis on Dielectric Boundary Effects of Modified Poisson--Nernst--Planck Equations."** *SIAM Journal on Applied Mathematics 78, no. 3 (2018): 1802-22.*

**Qiao, Zhonghua, Zhenli Xu, Qian Yin, and Shenggao Zhou. "Structure-Preserving Numerical Method for Maxwell-Ampère Nernst-Planck Model."** *Journal of Computational Physics 475 (2023/02/15/ 2023): 111845.*

**Qiao, Zhonghua, Zhenli Xu, Qian Yin and Shenggao Zhou (2023). "A Maxwell–Ampère Nernst–Planck Framework for Modeling Charge Dynamics."** SIAM Journal on Applied Mathematics 83(2): 374-393.

# Important Applications
## in Biology

# Electron Flow in ATP Production



**Electron Transport Chain**

Electrons

Electrons

Intermembrane space

$H^+$   $H^+$   $H^+$

I   Q   III   IV

II

$2e-$

Mitochondrial matrix

Inner mitochondrial membrane

*Khan Academy (CC BY 3.0).*
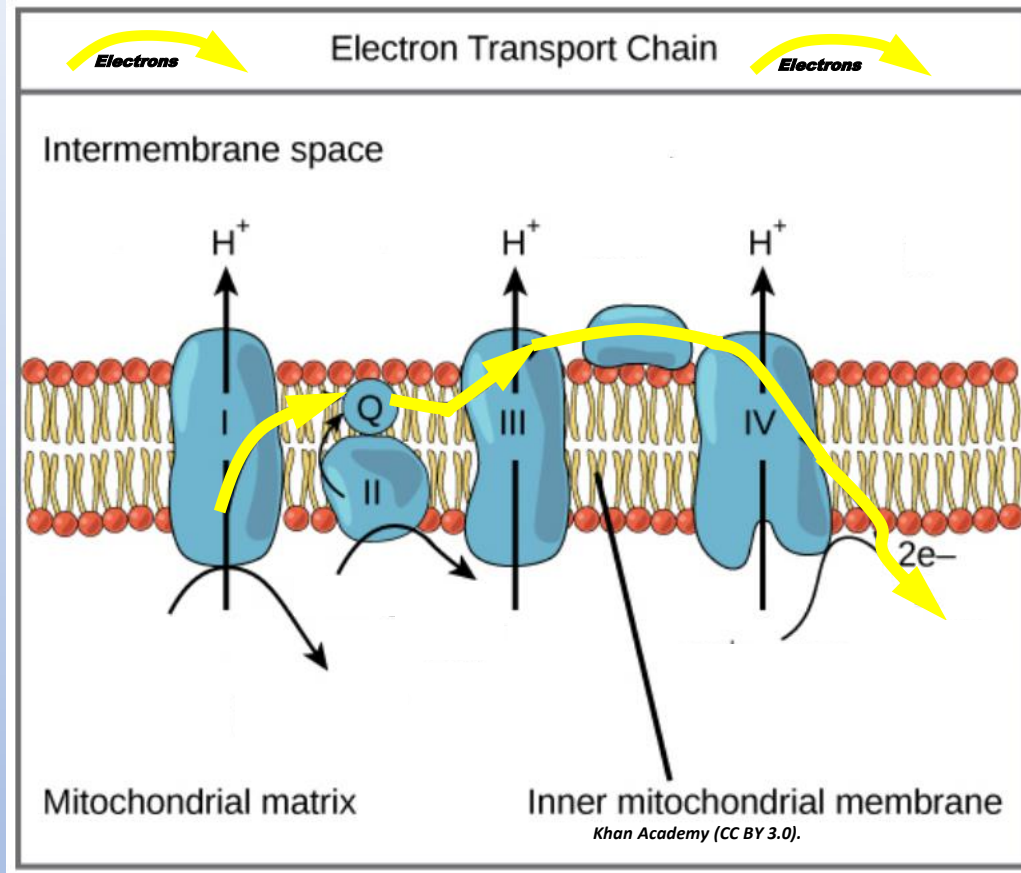
**Biologists have tried to compute Electron flow from Coulomb's law with little success since Nobel Prize to Peter Mitchell for Chemiosmotic Hypothesis 1978**

**In my view, Kirchhoff's law should be used.**
**We have started that process**
Xu,. Eisenberg, Song and Huang (2022). "Mathematical Model for Chemical Reactions in Electrolyte Applied to Cytochrome c  Oxidase: an **Electro-osmotic Approach**."
arXiv preprint arXiv:2207.02215.

# Some biologists have been Applying Maxwell to the Nerve Signal for a long time



Alan Hodgkin          William Rushton

Proc Roy Soc (London) Ser B. 1946;133:444-79.

# Channels are Chemically and Structurally INDEPENDENT

# Natural Function of  Channels Requires
# Coupling by the Electric Field



**Electric Field Couples Channels
so they can make a
Useful Electrical Signal
the Action Potential**

# Coupling in Natural Function is by Electric Field, i.e., VOLTAGE SPREAD

**Nerve Fiber with Insulating Myelin**

**Channels**



https://www.youtube.com/watch?v=tOTYO5WrXFU

**Na**          **K**

https://www.youtube.com/watch?v=oa6rvUJIg7o
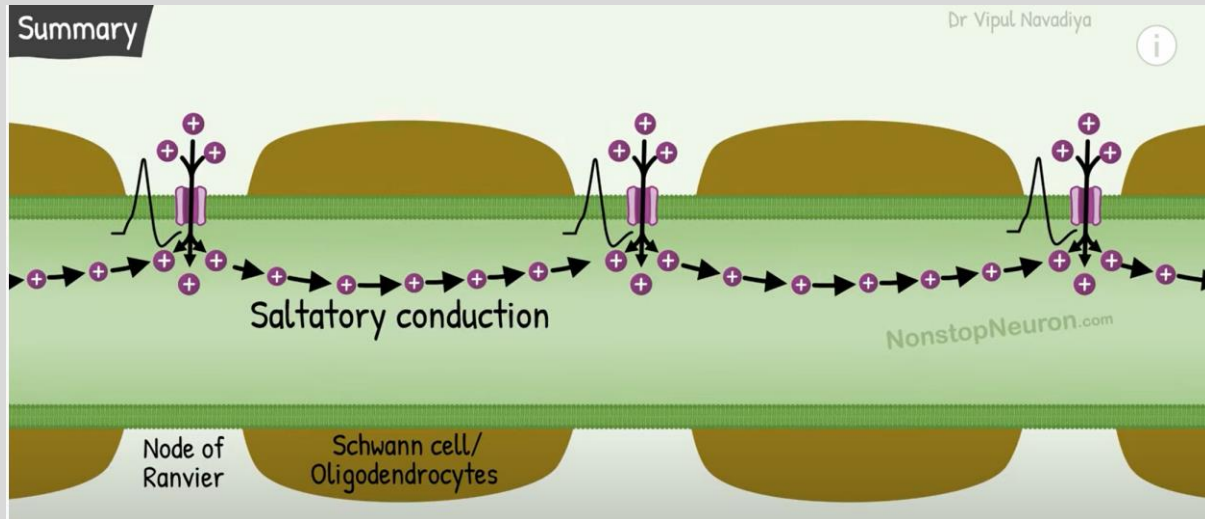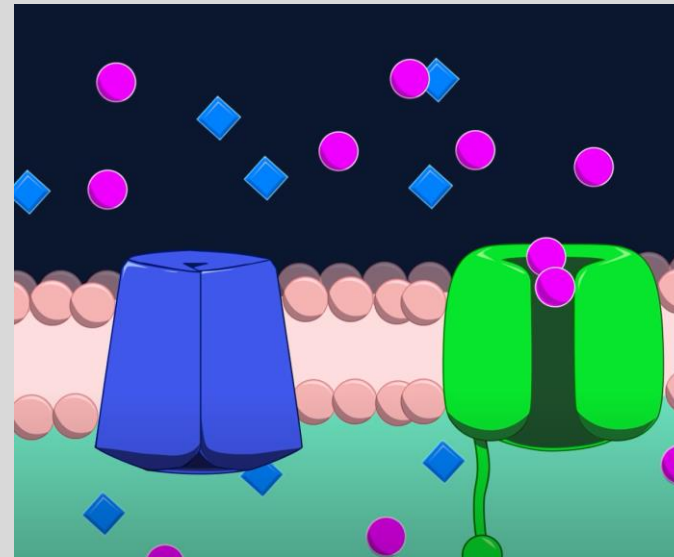
# Our Axopatch makes
# Voltage Clamp seem natural
## It is not.
# UNclamp is Natural !



AxoPatch 200B

Resistive feedback patch clamp

Axopatch 200B at room temp.

Axopatch 200B with active cooling



181 fS, pH 5.5

36 fS, pH 6.5

Cherny V et al. JGP 2003;121:615-628

25

# Any Questions?

# Displacement Current $\varepsilon_0 \partial \mathrm{E}/\partial t$ has a Special Place in Physics

## Charge has a Special Place in Physics

Displacement Current $\varepsilon_0 \, \partial \mathrm{E}/\partial t$ makes charge Lorentz Invariant

# Electrodynamics and Relativity

## **Charge has a Special Place in physics**

Charge <sub>on a particle (coulombs)</sub> does not vary

as it moves at velocity $v$,

even near the velocity of light $c$

Charge (coulombs) is "Lorentz Invariant"

Distance, time, and relativistic mass
## **Change Dramatically**
near the velocity of light as $\sqrt{1 - v^2/c^2}$

# Displacement Current $\varepsilon_0 \, \partial E/\partial t$ is a Property of Space according to Theory of Relativity

## Maxwell Equations and Relativity are almost the same thing

"**The special theory of relativity … was simply a systematic development of the electrodynamics of Clerk Maxwell and Lorentz**".

p. 57 of Einstein, A. 1934. *Essays in science,*
*originally published as Mein Weltbild 1933,*
*translated from the German by Alan Harris. Open Road Media.*

# Fundamental Result of Physics

$$\varepsilon_0 \, \frac{\partial \mathbf{E}}{\partial t} \ \text{ is } \ \text{everywhere}$$

# Some Surprising Biophysical Consequences of Kirchhoff Field Law

# Biophysical Consequences
# Of
# Perfect Conservation of Total Current

1. Total Current does not hop in Channels
2. Kirchhoff Coupling in Nerve Signal
3. Kirchhoff Coupling in Mitochondria and Transporters
4. Kirchhoff Coupling is different in vesicles/mitochondria, and in voltage clamped bilayers.

**Contradicts Intuition**

# Small Systems
# REQUIRE
# Continuum Description
## of
## Electric Current

**Because of $\varepsilon_0 \, \partial E / \partial t$**

## Current does NOT flow by hopping

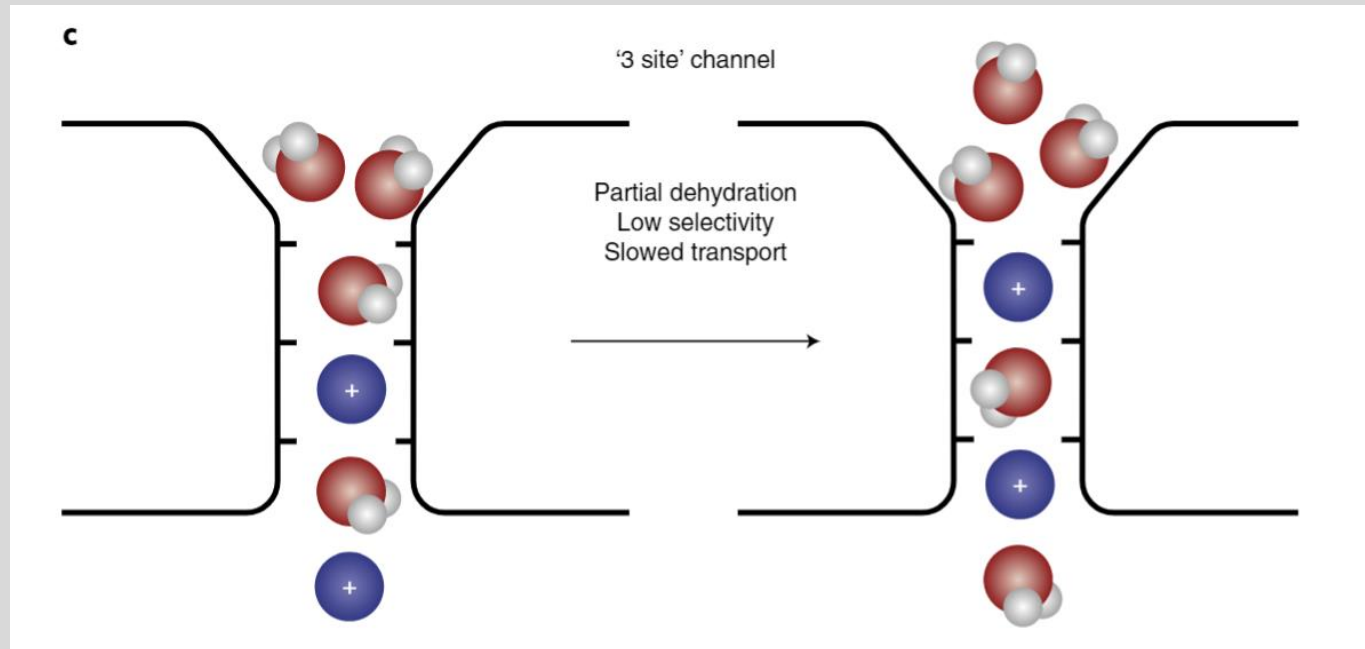Current is independent of location in series systems

**Particles can hop, total current cannot**

# What does this mean for Ion Channels?

## Knock On and Knock Off of Ions
### is

## IRRELEVANT for the Total Current $J_{total}$ Through the Channel

**Paradigm Change**



**Eisenberg (2020)**
**'Electrodynamics Correlates Knock-on and Knock-off: Current is Spatially Uniform in Ion Channels.'**
**Preprint on arXiv at 2002.09012**

Corry (2018) 'The naked truth about K$^+$ selectivity'. Nature Chemistry 10:799-800.

**View of Channels has been focused on movements of individual ions in channels,**

**But**

**Total Current $J_{total}$ is equal everywhere in a one dimensional channel**

*Paradigm Change*

# Position does <u>not</u> appear in equations for total current $J_{total}$ in a one dimensional channel

**References and Proofs in**
Eisenberg (2019) **Kirchhoff's Law can be Exact**. arXiv: 1905.13574

Eisenberg, Gold, Song, and Huang (2018)
**What Current Flows Through a Resistor?**
arXiv:1805.04814

# Some Surprising Physical Consequences of Kirchhoff Field Law

**Conservation of Current is Exact and Universal**
*So what?*

**(1) Current must always be described by Continuum Equations**

<span style="color:red">Because of $\varepsilon_0\, \partial E/\partial t$</span>

**(2) Particle motion does NOT define Current**

<span style="color:red">Contradicts Intuition</span>

**Current ≠ Flux of charge**

**Contradicts Intuition**

# Small Systems
# REQUIRE

# Continuum Description

## of

## Electric Current

**Because of $\varepsilon_0\ \partial E/\partial t$**

## Current does NOT flow by hopping

Current is independent of location in series systems

**Particles can hop, total current cannot**

**Total Current flow $J_{total}$ is equal everywhere
in a one dimensional system**

# Thermal Motion in Space does <u>not</u> appear in equations for flow of total current $J_{total}$ in a one dimensional system

**Thermal motion appears ONLY in time**

**What does this mean for Mathematical Models?**

**The image of total current flow $\mathbf{J}_{total}$ is very different VERY SMOOTH in space**

**Total Current $\mathbf{J}_{total}$ does not vary in space so**
**<u>Spatial Derivatives are not needed to describe total current</u>**

**But they are needed to describe everything else.**

**Spatial Variable does NOT appear in description of current in a one dimensional channel**

How take advantage of this enormous simplication?