Wigner Transport in Magnetic Fields

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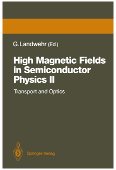
Keywords and Keypictures

- Gauge-invariant Wigner theory
- Kinetic and canonical momentum
- Quantum nonlocality
- Inhomogeneous (linear) magnetic field
- Interplay of electric and magnetic fields:

'A Wigner Function Study of Magnetotunneling'

N. Kluksdahl, A. Kriman, D. K. Ferry

Springer Series in Solid State Sciences, V87, 1989



AND

Outline

- Gauge-Invariant Wigner Formalism in terms of general EM fields (no potentials), but complicated
- Physical Settings: 2D Transport in (x, y), General $\mathbf{E}(x, y)$, Linear $\mathbf{B}(x, y)$, stationary fields
- Simulation Analysis: Interplay Between Electric and Magnetic Fields
- Local Effect of the Interplay between $\mathbf{E}$ and $\mathbf{B}$
- Nonlocal Effect of the Interplay between $\mathbf{E}$ and $\mathbf{B}$

Gauge-Invariant Wigner Formalism

Kinetic Momentum	Canonical Momentum	Hamiltonian	EM Fields and Potentials
$\mathbf{P}$	$\mathbf{p} = \mathbf{P} + e\mathbf{A}(\mathbf{x})$	$H = \frac{(\mathbf{p} - e\mathbf{A}(\mathbf{x}))^2}{2m} + e\phi$	$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$
physical quantity	mathematical quantity	$\mathbf{A}, \phi$ - EM potentials	$\mathbf{B} = \nabla \times A$

Density matrix

$$\rho(x + \tfrac{s}{2}, x - \tfrac{s}{2}, t) = \psi(x + \tfrac{s}{2})\psi(x - \tfrac{s}{2})^*$$

Evolution equation

$$\frac{\partial \rho(\mathbf{x} + \frac{\mathbf{s}}{2}, \mathbf{x} - \frac{\mathbf{s}}{2})}{\partial t} = \frac{1}{i\hbar}[H\rho]$$

WST

Weyl-Stratonovich Transform (WST)

$$f_w(\mathbf{P_m}, \mathbf{x}) = \int_{-\mathbf{L}/2}^{\mathbf{L}/2} \frac{d\mathbf{s}}{\mathbf{L}} e^{-\frac{i}{\hbar}\mathbf{s}\cdot\mathbf{P_m} - \frac{i}{\hbar}\frac{e}{2}\mathbf{s}\cdot\int_{-1}^{1} d\tau \mathbf{A}(\mathbf{x} + \frac{\mathbf{s}\tau}{2})} \rho(\mathbf{x} + \tfrac{\mathbf{s}}{2}, \mathbf{x} - \tfrac{\mathbf{s}}{2})$$

WST $\rightarrow$ Weyl Transform (WT) if $\mathbf{A} = 0$

WT with $\mathbf{p}$ introduces gauge dependent theory

WST removes the gauge dependence introducing $\mathbf{P}$

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{P_M}}{m}\cdot\frac{\partial}{\partial \mathbf{x}}\right) f_w(\mathbf{P_M}, \mathbf{x}) =$$
$$\sum_{\mathbf{m}=-\infty}^{\infty} \left\{ \frac{e}{2i\hbar}\int_{-1}^{1} d\tau D^F(\mathbf{x}, \mathbf{m}, \tau) - \frac{e}{2m}\int_{-1}^{1} d\tau \frac{\tau}{2} H^F(\mathbf{x}, \mathbf{m}, \tau)\cdot\frac{\partial}{\partial \mathbf{x}} \right.$$
$$\left. + \frac{e}{2i\hbar}\int_{-1}^{1} d\tau H^F(\mathbf{x}, \mathbf{m}, \tau)\cdot\frac{\mathbf{P_M}}{m} + \frac{e^2}{4mi\hbar}\int_{-1}^{1}\int_{-1}^{1} d\tau d\eta \frac{\tau}{2} I^F(\mathbf{x}, \mathbf{m}, \tau, \eta) \right\} f_w(\mathbf{P_{M-m}}, \mathbf{x})$$

In terms of the kinetic momentum $\mathbf{P}$

$$D^F(\mathbf{x}, \mathbf{m}, \tau) = -\int_{-\mathbf{L}/2}^{\mathbf{L}/2} \frac{d\mathbf{s}'}{\mathbf{L}} e^{-\frac{i}{\hbar}\mathbf{m}\Delta\mathbf{p}\mathbf{s}'} \left(\mathbf{s}'\cdot\mathbf{E}(\mathbf{x} + \frac{\mathbf{s}'\tau}{2})\right)$$

$$H^F(\mathbf{x}, \mathbf{m}, \tau) = \int_{-\mathbf{L}/2}^{\mathbf{L}/2} \frac{d\mathbf{s}'}{\mathbf{L}} e^{-\frac{i}{\hbar}\mathbf{m}\Delta\mathbf{p}\mathbf{s}'} \left[\mathbf{s}'\times\mathbf{B}(\mathbf{x} + \frac{\mathbf{s}'\tau}{2})\right]$$

$$I^F(\mathbf{x}, \mathbf{m}, \tau, \eta) = \int_{-\mathbf{L}/2}^{\mathbf{L}/2} \frac{d\mathbf{s}'}{\mathbf{L}} e^{-\frac{i}{\hbar}\mathbf{m}\Delta\mathbf{p}\mathbf{s}'} \left(\mathbf{s}'\times\mathbf{B}(\mathbf{x} + \frac{\mathbf{s}'\eta}{2})\right)\cdot\left(\mathbf{s}'\times\mathbf{B}(\mathbf{x} + \frac{\mathbf{s}'\tau}{2})\right)$$

Quantities depending on $\mathbf{E}, \mathbf{B}$ of general form (no EM potentials)

$\mathbf{P_m} = \mathbf{m}\Delta\mathbf{P}, \quad \mathbf{m} \in \mathcal{N}, \quad \Delta\mathbf{P} = \frac{2\pi\hbar}{\mathbf{L}}$ - discrete momentum for finite systems;

$\mathbf{L} \rightarrow \infty \quad \mathbf{P_m} \rightarrow \mathbf{P}$ continuous momentum - long coherence length limit

Gauge-Invariant Wigner Formalism

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{P_M}}{m}\cdot\frac{\partial}{\partial \mathbf{x}}\right)f_w(\mathbf{P_M},\mathbf{x})=\sum_{\mathbf{m}=-\infty}^{\infty}\left\{\underbrace{\frac{e}{2i\hbar}\int_{-1}^{1}d\tau D^F(\mathbf{x},\mathbf{m},\tau)}_{A}-\underbrace{\frac{e}{2m}\int_{-1}^{1}d\tau\frac{\tau}{2}H^F(\mathbf{x},\mathbf{m},\tau)\cdot\frac{\partial}{\partial\mathbf{x}}}_{B}\right.$$

$$\left.+\underbrace{\frac{e}{2i\hbar}\int_{-1}^{1}d\tau H^F(\mathbf{x},\mathbf{m},\tau)\cdot\frac{\mathbf{P_M}}{m}}_{C}+\underbrace{\frac{e^2}{4mi\hbar}\int_{-1}^{1}\int_{-1}^{1}d\tau d\eta\frac{\tau}{2}I^F(\mathbf{x},\mathbf{m},\tau,\eta)}_{D}\right\}f_w(\mathbf{P_{M-m}},\mathbf{x}) \quad (1)$$

$$D^F(\mathbf{x},\mathbf{m},\tau)=-\int_{-\mathbf{L}/2}^{\mathbf{L}/2}\frac{d\mathbf{s}'}{\mathbf{L}}e^{-\frac{i}{\hbar}\mathbf{m}\Delta\mathbf{p}\mathbf{s}'}\left(\mathbf{s}'\cdot\mathbf{E}(\mathbf{x}+\frac{\mathbf{s}'\tau}{2})\right)$$

$$H^F(\mathbf{x},\mathbf{m},\tau)=\int_{-\mathbf{L}/2}^{\mathbf{L}/2}\frac{d\mathbf{s}'}{\mathbf{L}}e^{-\frac{i}{\hbar}\mathbf{m}\Delta\mathbf{p}\mathbf{s}'}\left[\mathbf{s}'\times\mathbf{B}(\mathbf{x}+\frac{\mathbf{s}'\tau}{2})\right]$$

$$I^F(\mathbf{x},\mathbf{m},\tau,\eta)=\int_{-\mathbf{L}/2}^{\mathbf{L}/2}\frac{d\mathbf{s}'}{\mathbf{L}}e^{-\frac{i}{\hbar}\mathbf{m}\Delta\mathbf{p}\mathbf{s}'}\left(\mathbf{s}'\times\mathbf{B}(\mathbf{x}+\frac{\mathbf{s}'\eta}{2})\right)\cdot\left(\mathbf{s}'\times\mathbf{B}(\mathbf{x}+\frac{\mathbf{s}'\tau}{2})\right)$$

- Complicated for $\mathbf{E}$, $\mathbf{B}$ fields of general spatio-temporal dependence
- Lack of any computational experience
- CONSIDER simplifying physical settings
- CLUE: For homogeneous $\mathbf{B}$ (1) takes the standard form with magnetic force on the left
  A $\rightarrow$ Wigner potential; B,D $\rightarrow$ 0; C $\rightarrow$ magnetic component of Lorentz force ($\mathbf{L}\rightarrow\infty$)
- CONSIDER the next term in Taylor series of $\mathbf{B}$: linear $\mathbf{B}(x,y)$
  2D Transport in (x,y) plane, general $\mathbf{E}(x,y)$, (stationary fields)

Physical Settings: 2D Transport, Linear $\mathbf{B}$, General $\mathbf{E}$

$\mathbf{E}(x,y) \in (x,y)$ plane $\to$ Wigner potential | $\mathbf{B} = (0,0,B_0 + B_1 y) \perp (x,y)$ plane | in the limit $\mathbf{L} \to \infty$; | D neglected

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{P}}{m}\cdot\frac{\partial}{\partial \mathbf{x}} + \frac{\mathbf{P}\times\mathbf{B}(y)}{m}\cdot\frac{\partial}{\partial \mathbf{P}}\right) f_w(\mathbf{P},\mathbf{x}) =$$ Liouville Operator (LO)

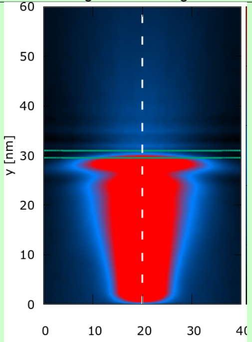
$$\int d\mathbf{P}' V_w(\mathbf{P}-\mathbf{P}',\mathbf{x}) f_w(\mathbf{P}',\mathbf{x}) +$$ Wigner Potential (WP)

$$\frac{B_1\hbar^2}{m}\frac{e}{12}\left(\frac{\partial^2}{\partial P_y^2}\frac{\partial}{\partial x} - \frac{\partial}{\partial P_x}\frac{\partial}{\partial P_y}\frac{\partial}{\partial y}\right) f_w(\mathbf{P},\mathbf{x})$$ High order f_w Derivatives (HoD)

Simulation Setup

- Analyze interplay between LO and WP
- Magnetic fields $\mathbf{B}(y) = B_0 + B_1 y$ with $B_0 \gg B_1$: HoD neglected
- Gaussian Wigner state with kinetic energy $0.1eV$ and $\sigma_{x,y} = 3nm$
- Injected from the bottom towards $0.3eV$, $1nm$ barrier (green lines), tunneling only
- Dashed line indicates the mean path of the state's evolution.
- Figure: The 2D electron density $n(x,y)$ resembles the *tongue* of *The Rolling Stones*

Magnetotunneling



Simulations: Local Effect on Tunneling

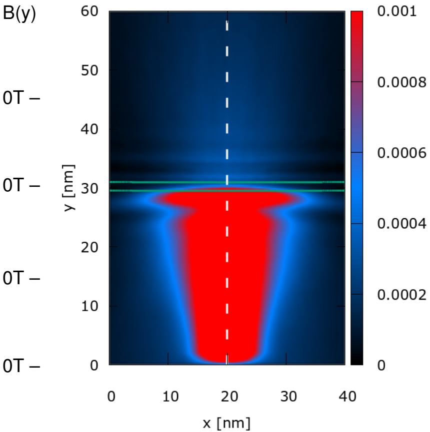


Figure: $B_0 = 0$, $B_1 = 0$. Without magnetic field the electron density $n(x, y)$ reflects the symmetry of the task. Fine oscillations are observed above the barrier.

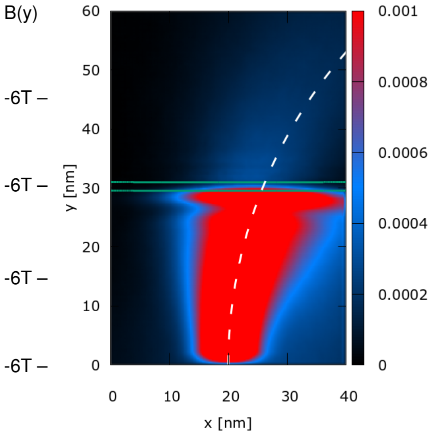


Figure: $B_0 = -6T$, $B_1 = 0$. A constant magnetic field bends the density and thus the mean path and destroys the oscillations above the barrier

Simulations: Local Effect on Tunneling

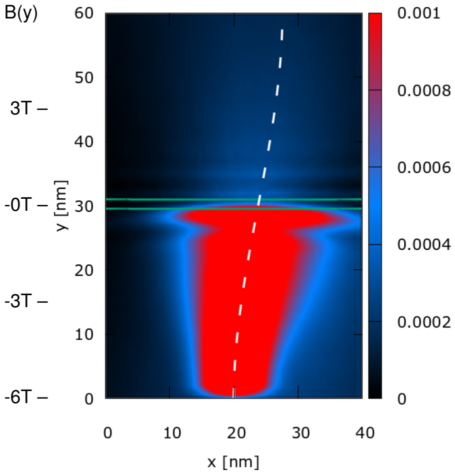


Figure: $B_0 = -6T$, $B_1 = 0.2T/nm$.

–The magnetic field becomes zero at the barrier: The oscillations appear again above it, in the upper half of the domain.

–The switch of the sign of $\mathbf{B}$ causes a snake type of mean path.

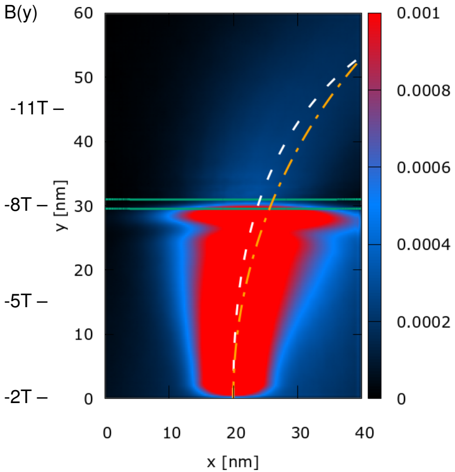
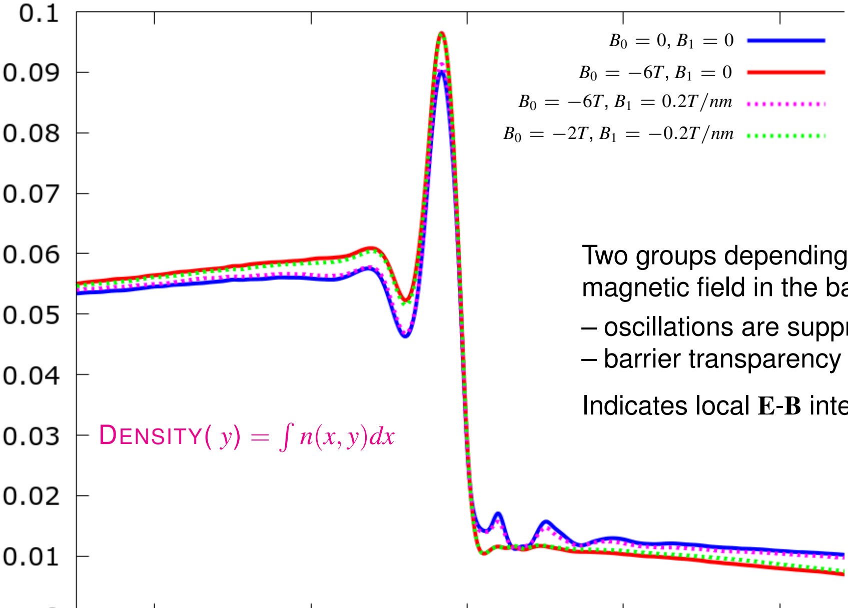


Figure: $B_0 = -2T$, $B_1 = -0.2T/nm$.

–The magnetic field is large at the barrier. The fine oscillations are suppressed as in the constant ($B_0 = -6T$) magnetic field case.

–The mean paths differ, but guide the state to same final point.

Local Effect on Tunneling



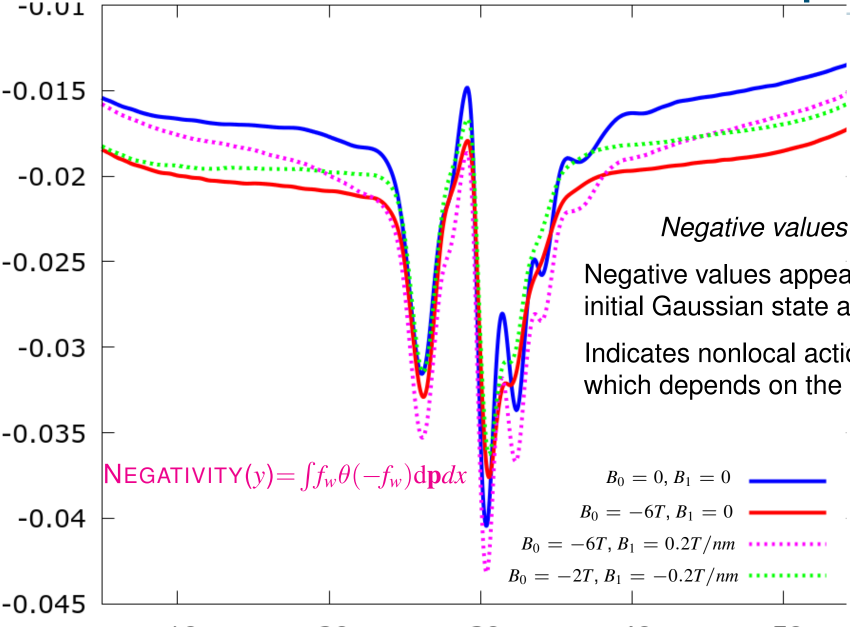
Two groups depending on the presence of magnetic field in the barrier region

– oscillations are suppressed

– barrier transparency is affected

Indicates local $\mathbf{E}$-$\mathbf{B}$ interaction

Nonocal Effect of the E-B Interplay



Negative values imply quantumness

Negative values appear in the injected positive initial Gaussian state away before the barrier

Indicates nonlocal action of the barrier which depends on the magnetic field

Conclusions

- The gauge-invariant Wigner equation - in terms of *general* EM fields, but: computationally challenging, numerical approaches yet not developed
- First experiences: 2D Transport, *general* $\mathbf{E}$, *linear* $\mathbf{B}$, *stationary* fields
- Equation linking magnetic aware Liouville, Wigner Potential and High-order Derivative operators (LO,WP,HoD). Simulation settings where HoD is small and neglected
- The interplay of the magnetic LO and WP suggest local and non-local effects, which need further analysis.
- Inhomogeneous magnetic field effects are potential candidates for control and manipulation of electron states.