# Deterministic approach for Skyrmionic Dynamics at Non-zero Temperatures

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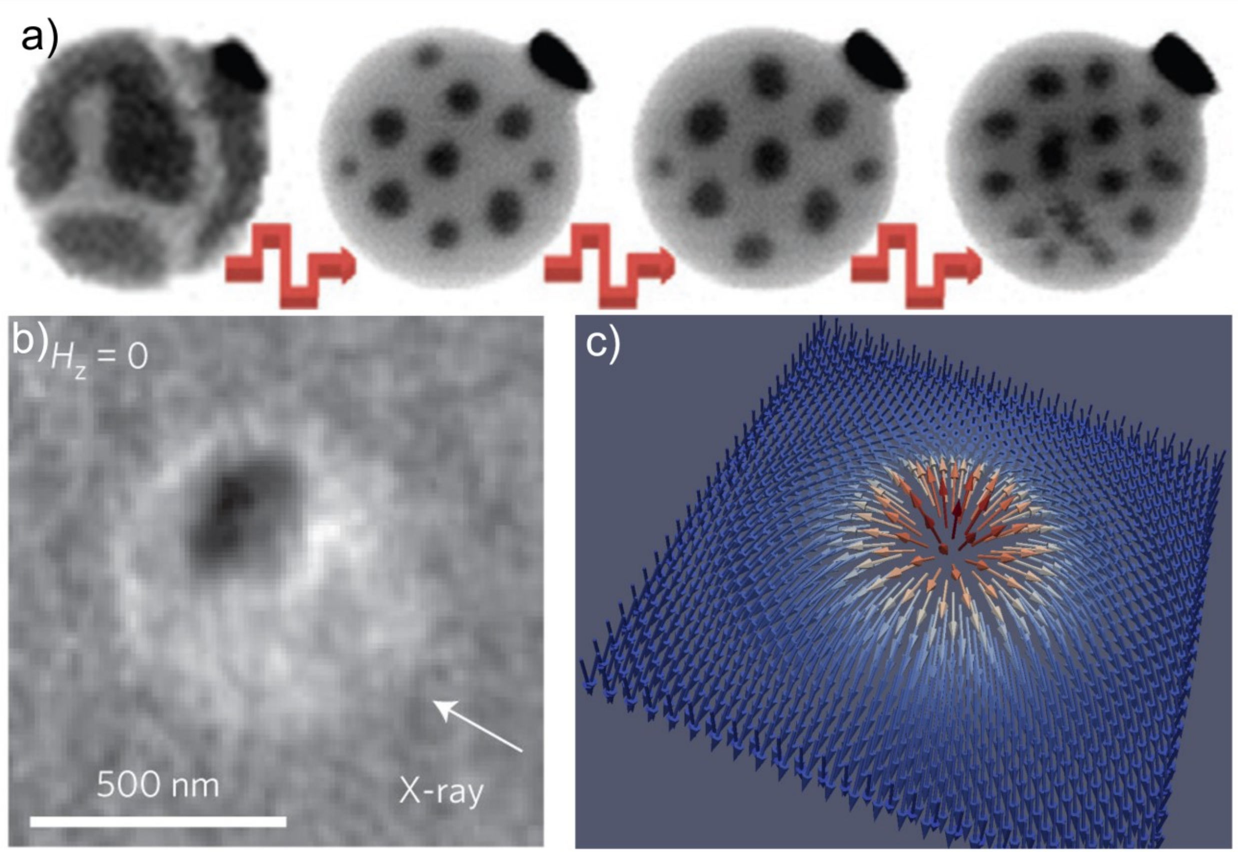
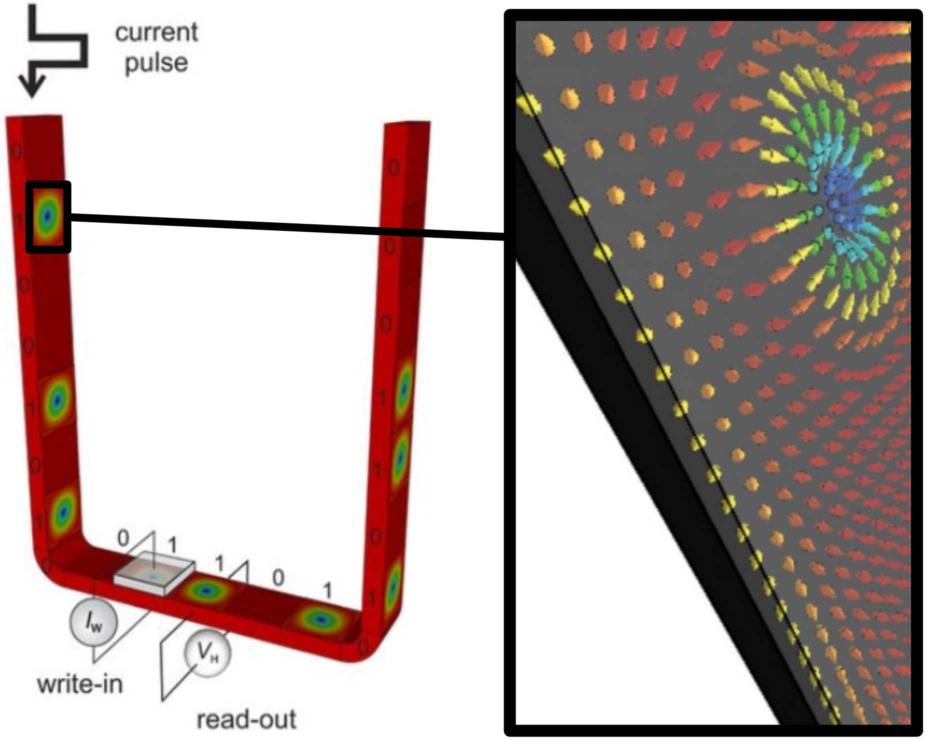
# Contents

# Motivation: Spintronics with skyrmions




a) S. Woo *Nature Materials* (**2016**)
b) O. Boulle *Nature Nanotechnology* (**2016**)

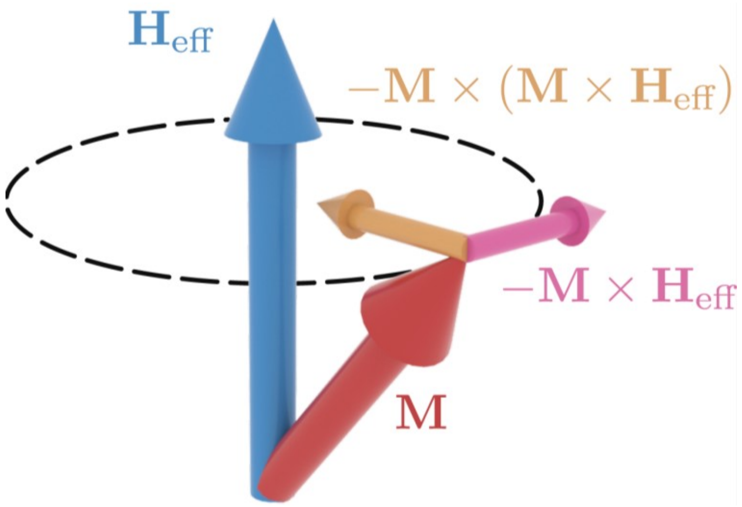



S. Zhang *Scientific Reports* (**2016**)

# LLG Equation

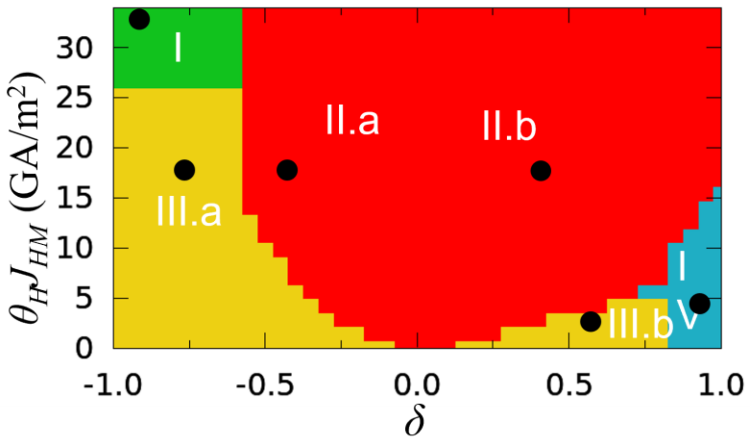
Laundau-Lifshitz-Gilbert equation determines the time evolution:

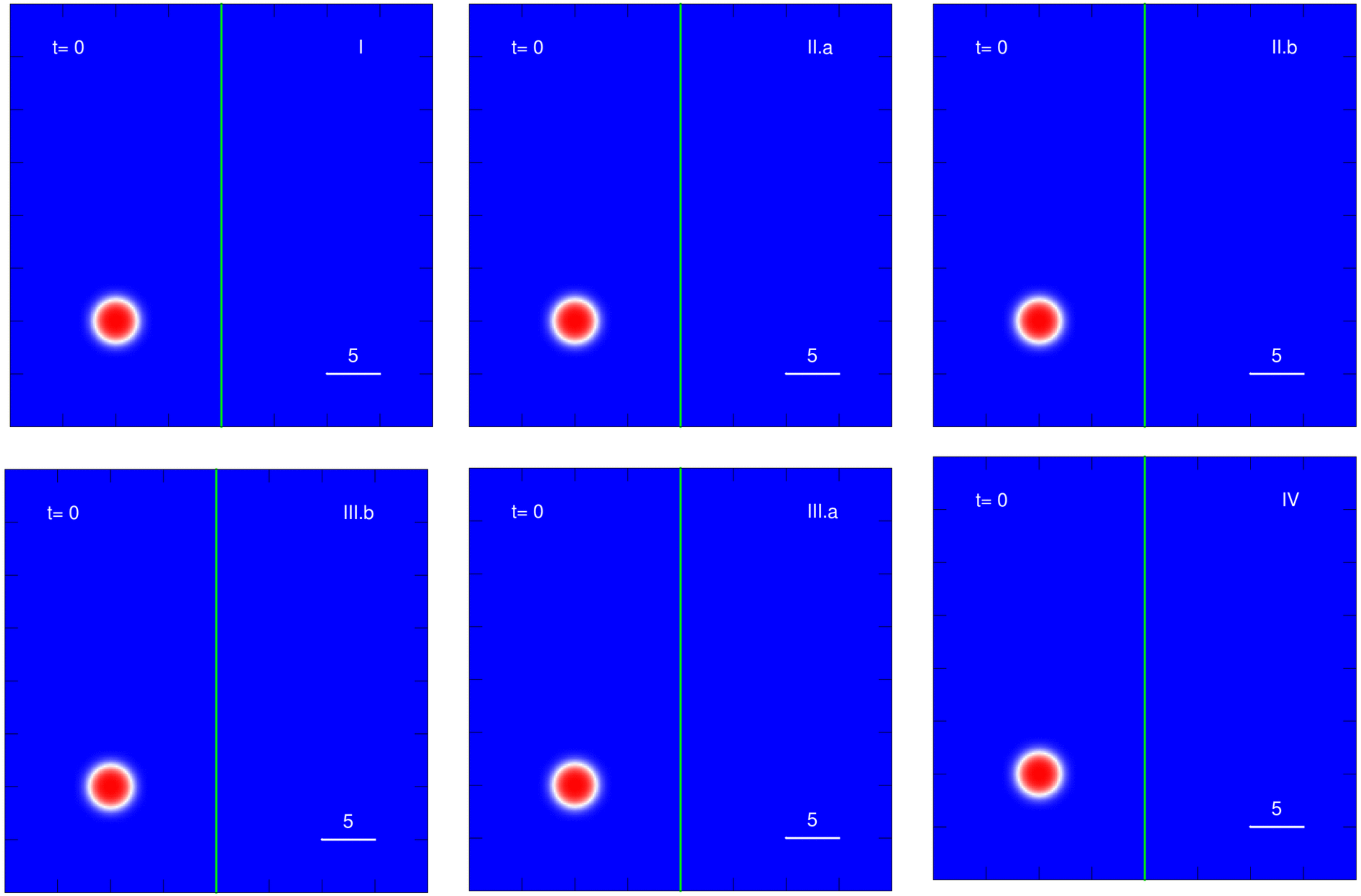
$$(1+\alpha^2)\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m}\times\mathbf{H}_{\text{eff}} - \frac{\gamma\alpha}{M_s}\mathbf{m}\times(\mathbf{m}\times\mathbf{H}_{\text{eff}}) + \mathbf{T}$$




$$\mathbf{H}_{\text{eff}} = \frac{2A_{\text{ex}}}{\mu_0 M_s}\nabla^2\mathbf{m} + \frac{2K_{\text{an}}}{\mu_0 M_s}m_z\hat{\mathbf{z}} + \mathbf{H}_{\text{a}} + \frac{2D_{\text{DM}}}{\mu_0 M_s}\left[(\nabla\mathbf{m})\,\hat{z} - \nabla m_z\right]$$

# Skyrmion – LD interaction



# Rigid Model

Micromagnetic model

Rigid  model

# Rigid Model: Basics

$$(\mathbb{G} - M_s\alpha\mathbb{D})\mathbf{v}_s + \gamma M_s^2\mathbf{F} = 0$$

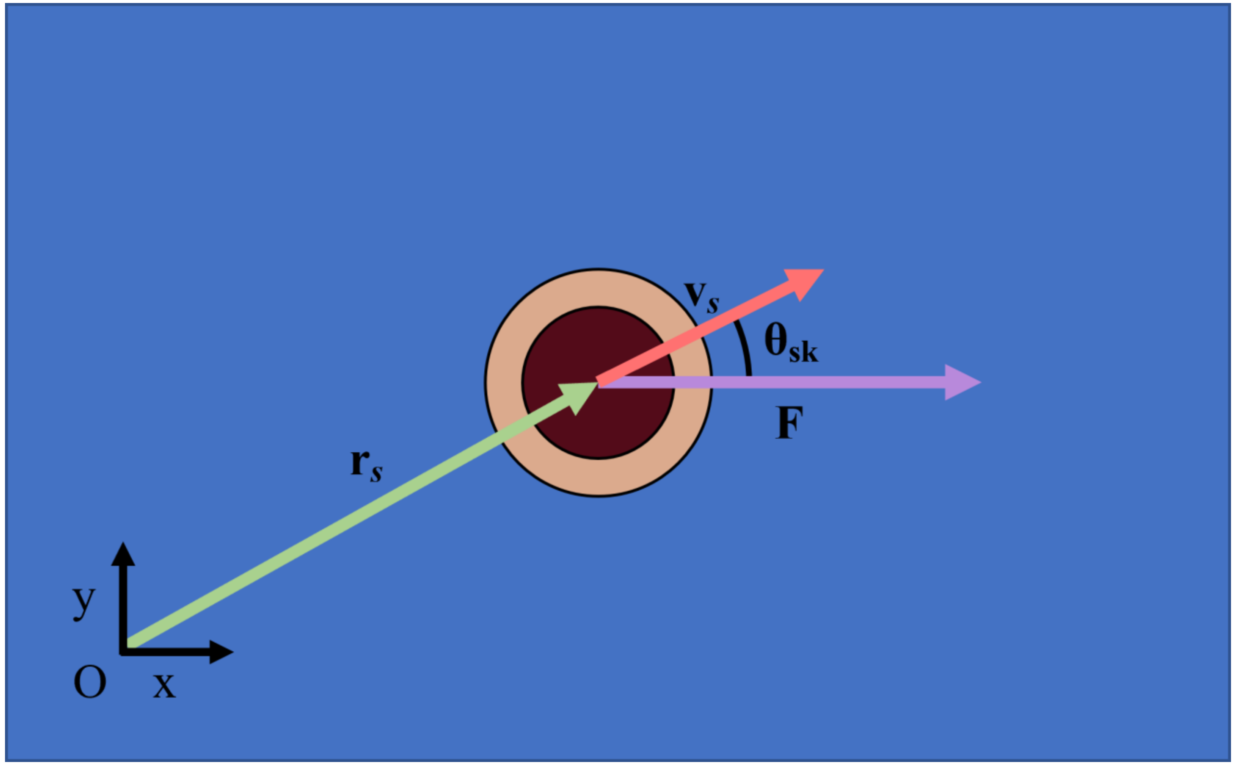
$$\mathbf{F}_{\text{SHE}} = M_s(\mathbb{N} + \zeta M_s\mathbb{Y})\mathbf{v}_H$$

$$\mathbf{v}_H = -\frac{\mu_B\theta_H}{|e|M_s}(\hat{\mathbf{z}} \times \mathbf{J}_H)$$

$$\mathbb{N}_{\nu\upsilon} = \frac{1}{d}\int_V \left(\frac{\partial \mathbf{M}_0}{\partial \nu} \times \mathbf{M}_0\right)_\upsilon \mathrm{d}V$$

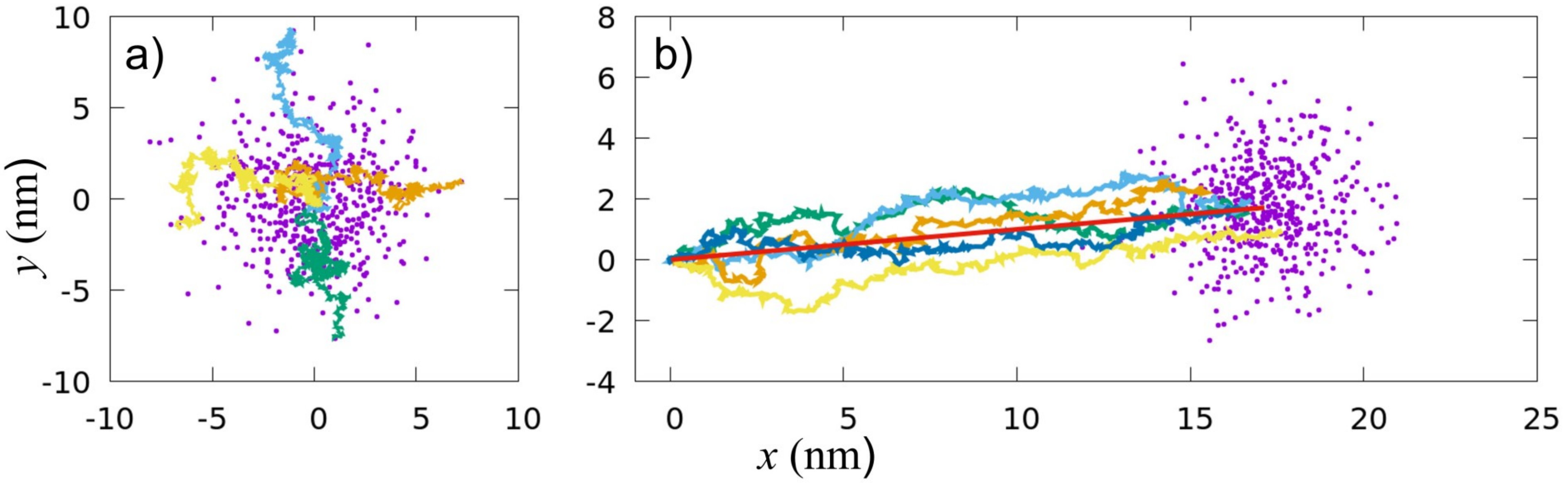
$$\mathbb{Y}_{\nu\upsilon} = \frac{1}{d}\int_V \frac{\partial(\mathbf{M}_0)_\upsilon}{\partial \nu}\mathrm{d}V'$$

$$(\mathbf{F}_{\text{ext}})_\nu = \int_V \mathbf{H}_{\text{ext}} \cdot \frac{\partial \mathbf{M}_0}{\partial \nu}\mathrm{d}V$$



# Temperature: Stochastic TE

$$(\mathbb{G} - M_s\alpha\mathbb{D})\mathbf{v}_s + M_s\mathbb{N}\mathbf{v}_H + \gamma M_s^2(\mathbf{F}_{\text{ext}} + \mathbf{F}_{\text{st}}) = 0$$



# Skyrmion Fokker-Planck Equation

$$\frac{\partial}{\partial t}p(\mathbf{r},t) = -\nabla\cdot[p(\mathbf{r},t)(\mathbf{V}_{drv}+\mathbf{V}_{ext})] + D_d\nabla^2 p(\mathbf{r},t)$$

$$D_d = \frac{\gamma M_s^3 \alpha D k_B T}{\mu_0(G^2 + D^2\alpha^2 M_s^2)}$$

$$\mathbf{V}_{drv} = -(\mathbb{G} - \alpha M_s \mathbb{D})^{-1} M_s \mathbb{N} \mathbf{V}_H$$

$$\mathbf{V}_{ext} = -(\mathbb{G} - \alpha M_s \mathbb{D})^{-1} \gamma M_s^2 \mathbf{F}_{ext}$$

- The Fokker-Planck equation is a **continuity equation**, hence the probability is conserved.

- The solution is the **probability density** *p*(**r**,*t*) of finding a skyrmion in a given position and time, a **deterministic magnitude**.

# Analytical Solution: Free skyrmion

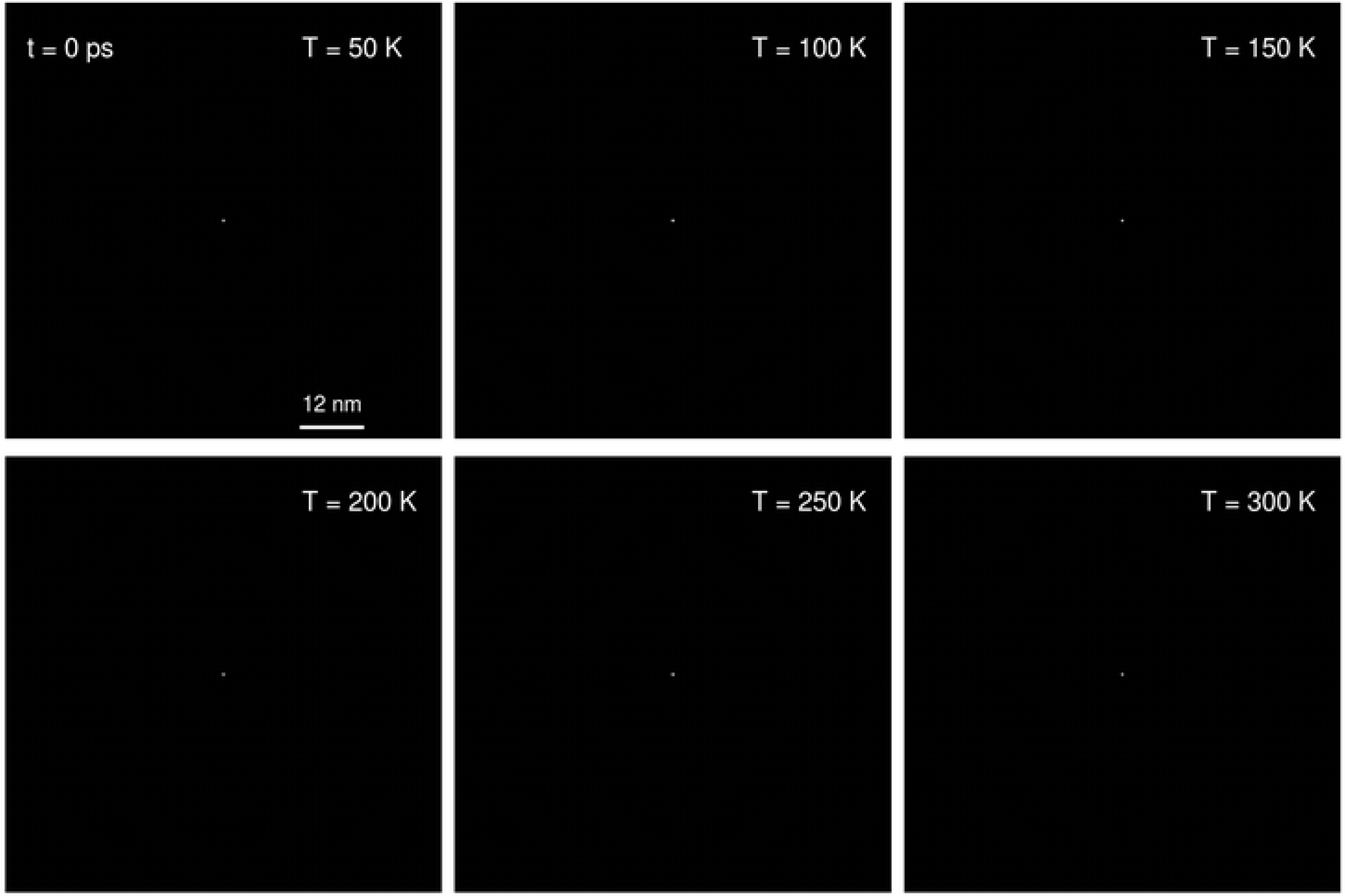
$$\mathbf{F}_{ext} = 0 \qquad \mathbf{V}_H = (0, V_y)$$

No external agents (apart from a driving current)

→ **brownian diffusion** that is **translated at constant speed.**

$$p(\mathbf{r}, t=0) = \mathcal{N}(0, \sigma)$$

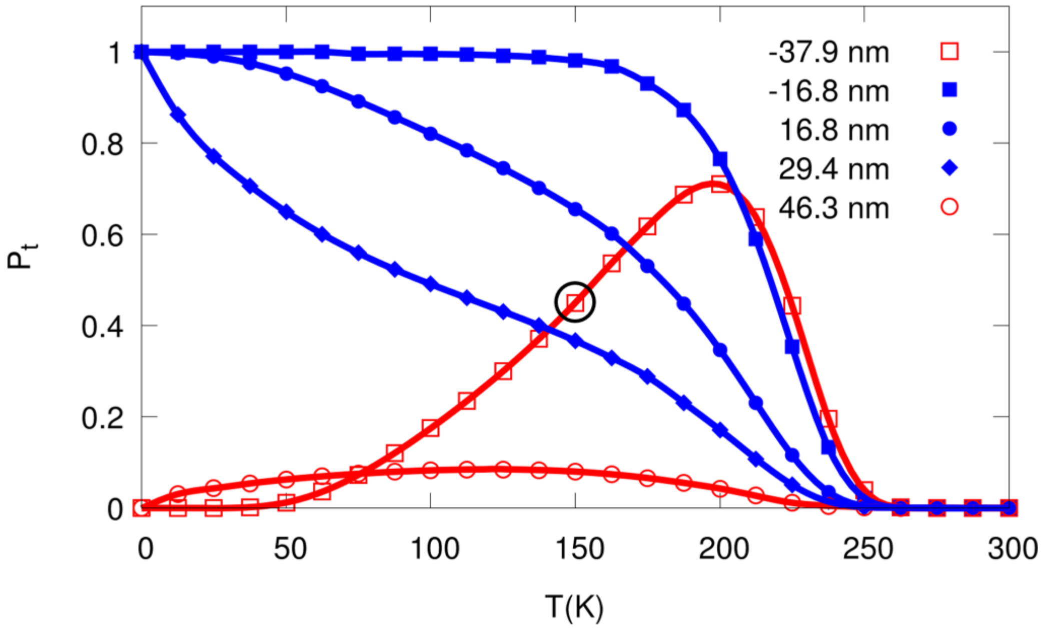
$$p(\mathbf{r}, t) = \mathcal{N}\left(\mathbf{r} - \mathbf{V}_{drv}t, \sqrt{\sigma^2 + 2D_d t}\right)$$

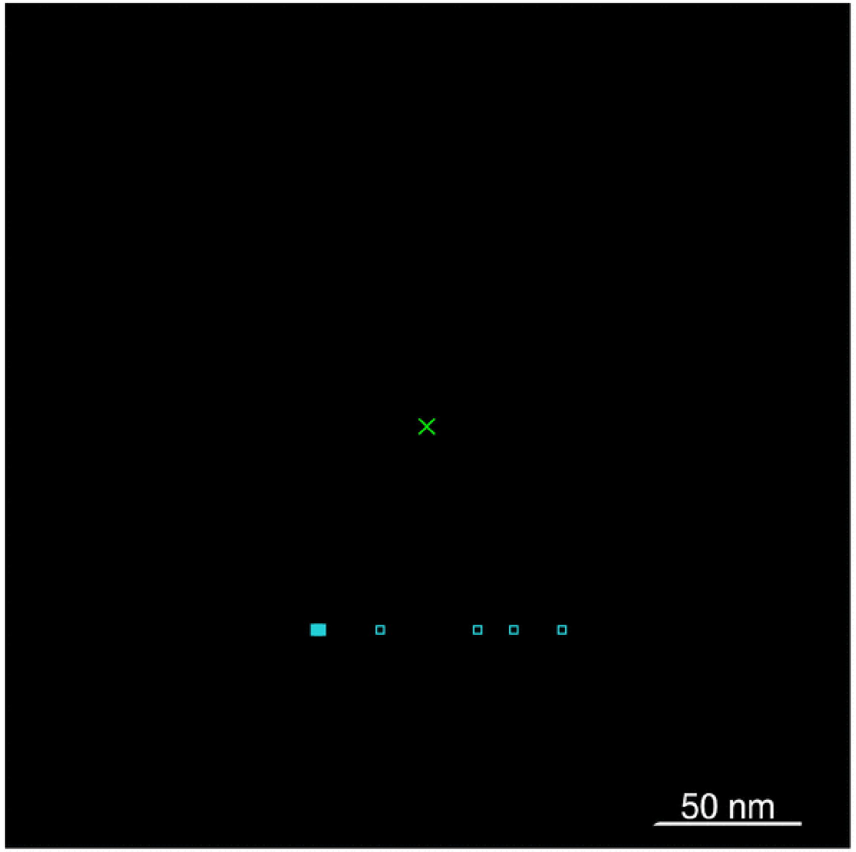


# Skyrmion dynamics under a Pin Potential

$$\mathbf{F}_{ext} = -F_{0p}\,\frac{\mathbf{r}}{\lambda}\exp\left(-\frac{|\mathbf{r}|^2}{\lambda^2}\right) \qquad \mathbf{V}_H = (0, V_y)$$

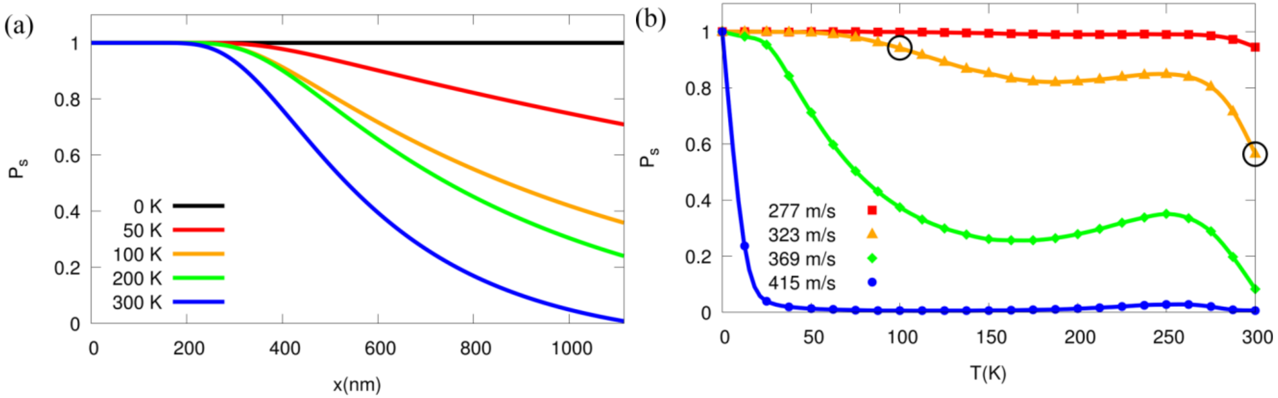
- If the temperature is 0, then skyrmions **will** or **will not** be trapped.
- If we have a non-zero temperature, skyrmions **may** or **may not** be trapped.



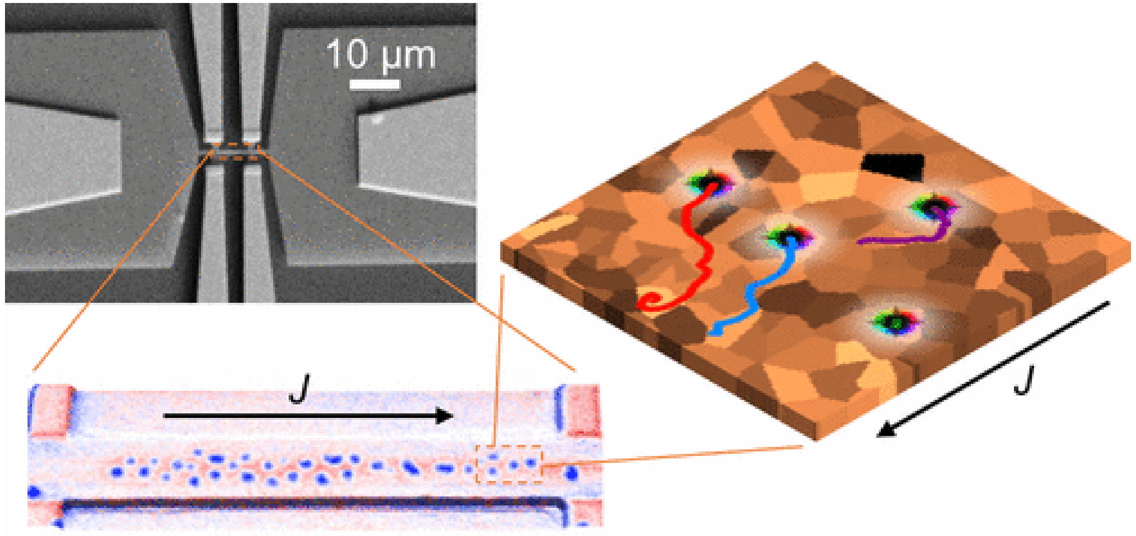



# Skyrmion dynamics in a Racetrack

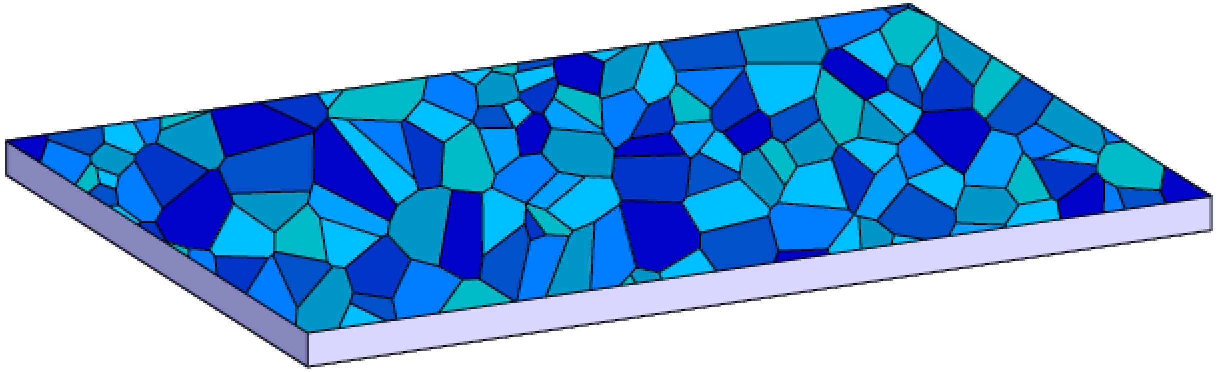
$$\mathbf{F}_{ext} = F_{0t}\left[-\mathrm{e}^{-\frac{W-y}{\lambda}} + \mathrm{e}^{-\frac{W+y}{\lambda}}\right]\hat{\mathbf{y}} \qquad \mathbf{V}_H = (0, -V_y)$$




# Racetrack with grains




W. Legrand *Nano Letters* (**2017**)



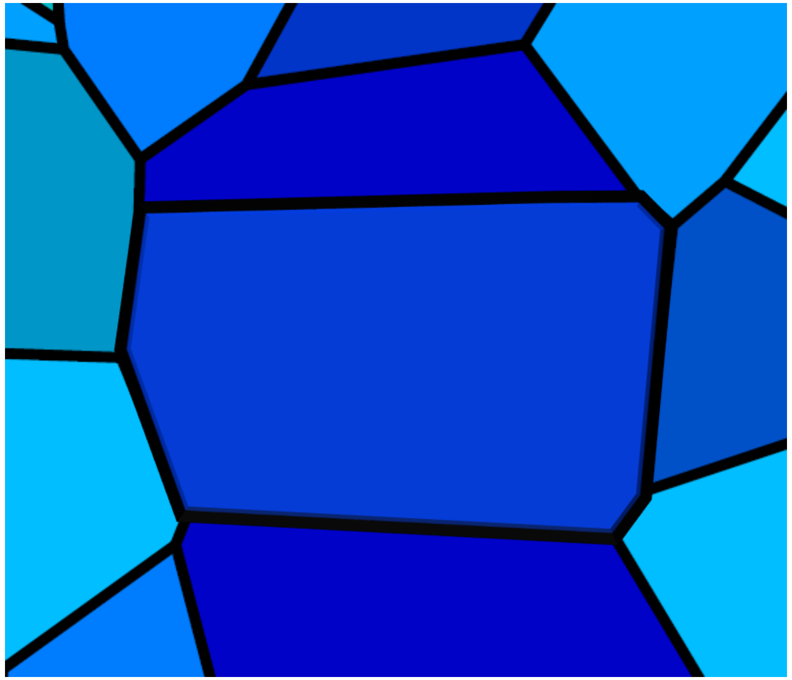
# 7. Modeling Granularity

$$\mathbf{F}_{\mathrm{ld}} = \frac{Ka^2\gamma}{\mu_0 M_s}\,\frac{4\,\Delta K\,R^2 r(R^2 - r^2)}{(R^2 + r^2)^3}\,\hat{\mathbf{r}}$$

$$\mathbf{F}_{\mathrm{gr}} = \int_{S_g} \sigma_{\mathrm{ld}}\,\mathbf{F}_{\mathrm{ld}}\,\mathrm{d}S \qquad \sigma_{\mathrm{ld}} = 1/a^2$$

$$f_G = (2K\Delta K\gamma R^2)/(\mu_0 M_s)$$

$$r_\pm = \sqrt{(r \pm L_g/2)^2 + R^2}$$



$$\mathbf{F}_{\mathrm{gr}} = f_G\left[\arctan\left(\frac{L_g/2}{r_0}\right)\left(\frac{-2}{r_0} + \frac{R^2}{r_0^3}\right) + \frac{R^2 L_g}{2r_0^2[(L_g/2)^2 + r_0^2]}\right]_{r_-}^{r_+}\hat{\mathbf{r}}$$

# 7. Modeling Granularity

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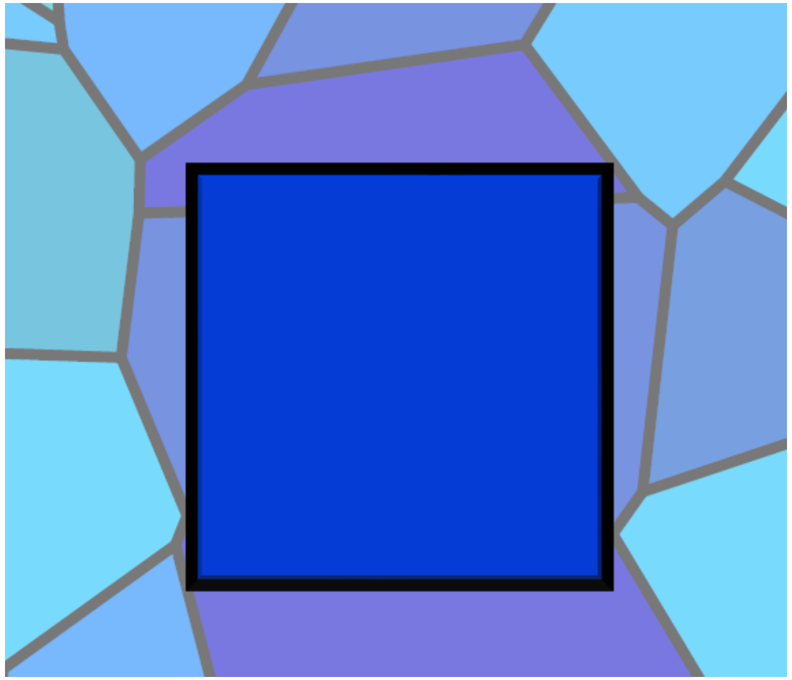

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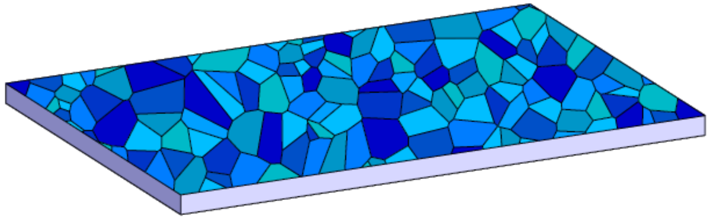
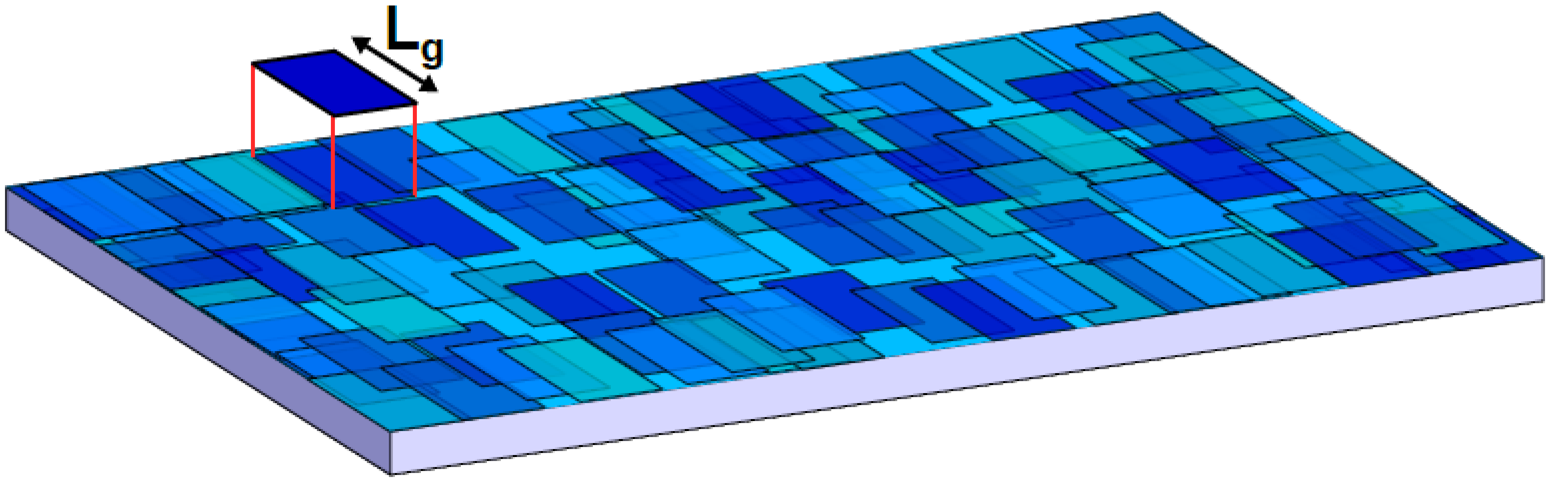


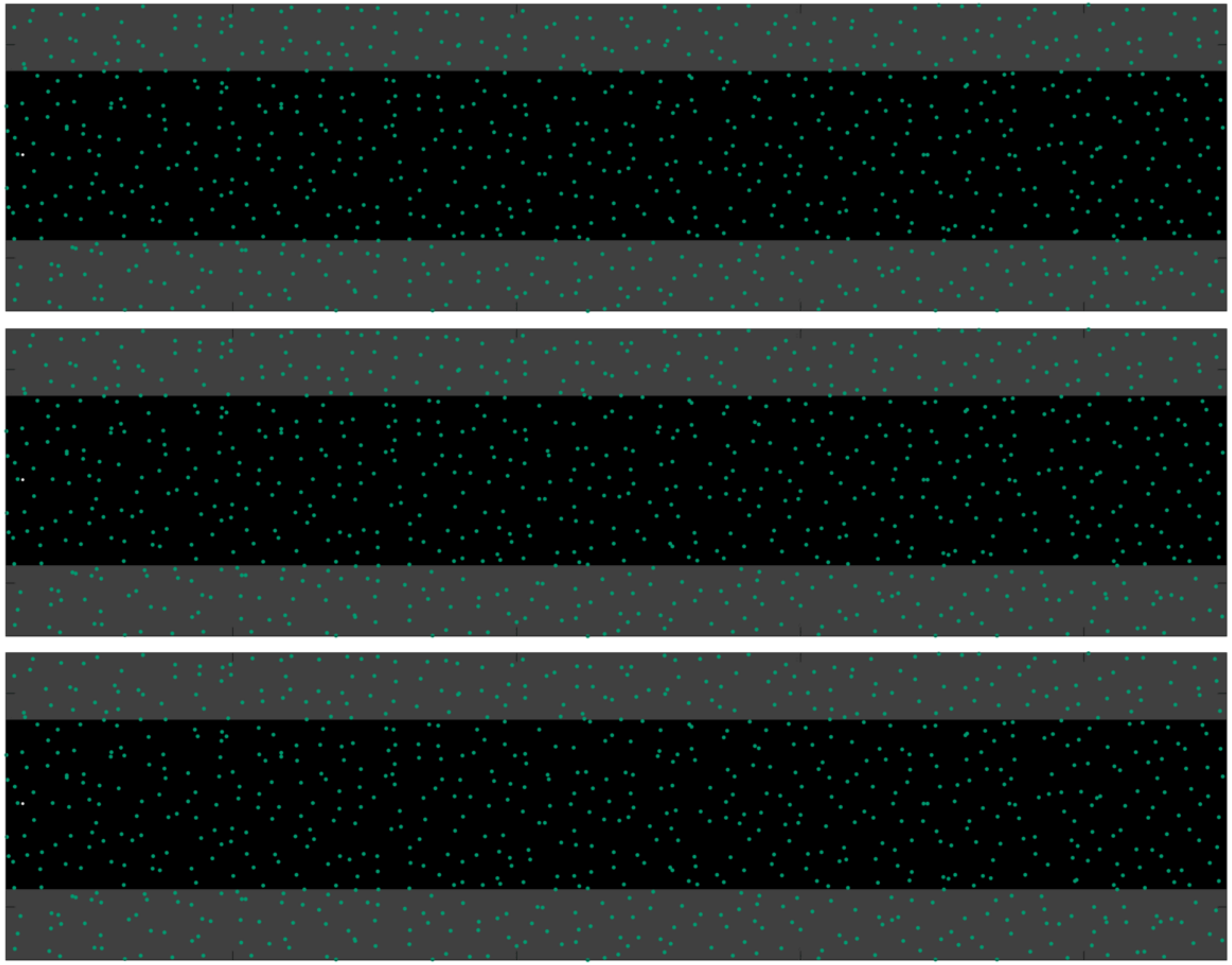


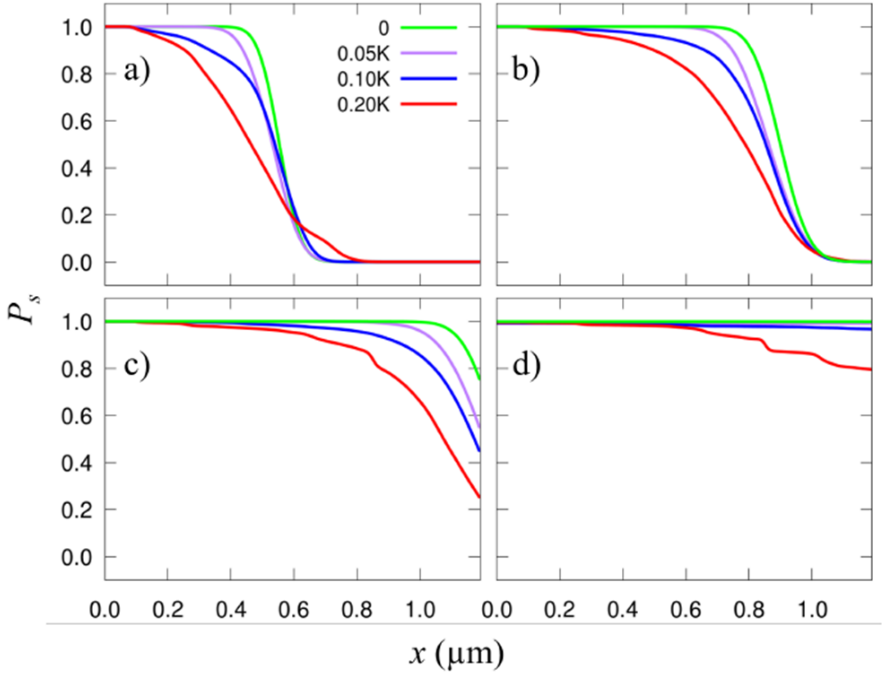
# Solution of the FPE

0%

10%

20%





# Conclusions

1. Granularity and temperature can be incorporated (simultaneiously) in the skyrmioninc modeling

2. The rigid approximation is valid for a realistic range of parametres

3. Depending on the granularity, the racetrack can prevent skyrmions to transport information at a desired precision

4. The detail of the shape of grains is less important than their heterogeneity

# Thank you for your attention

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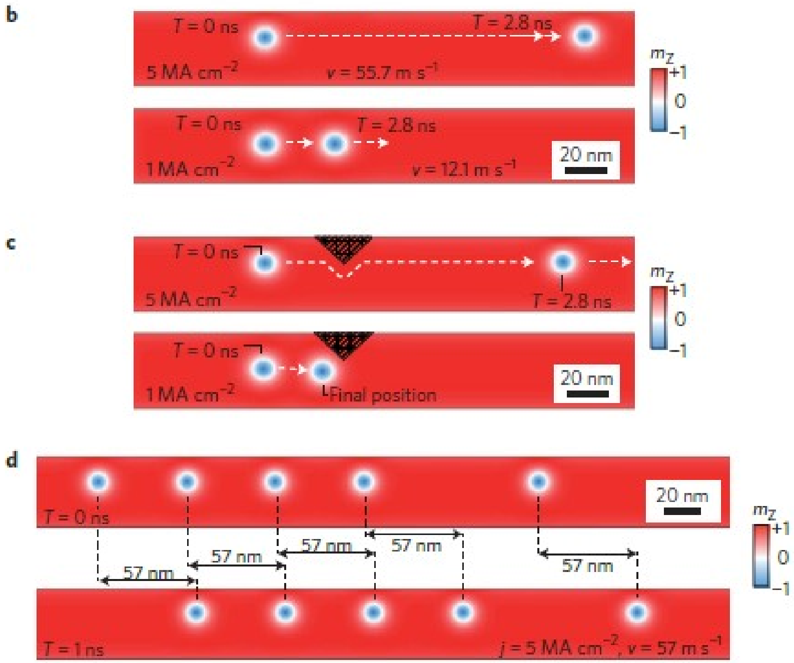
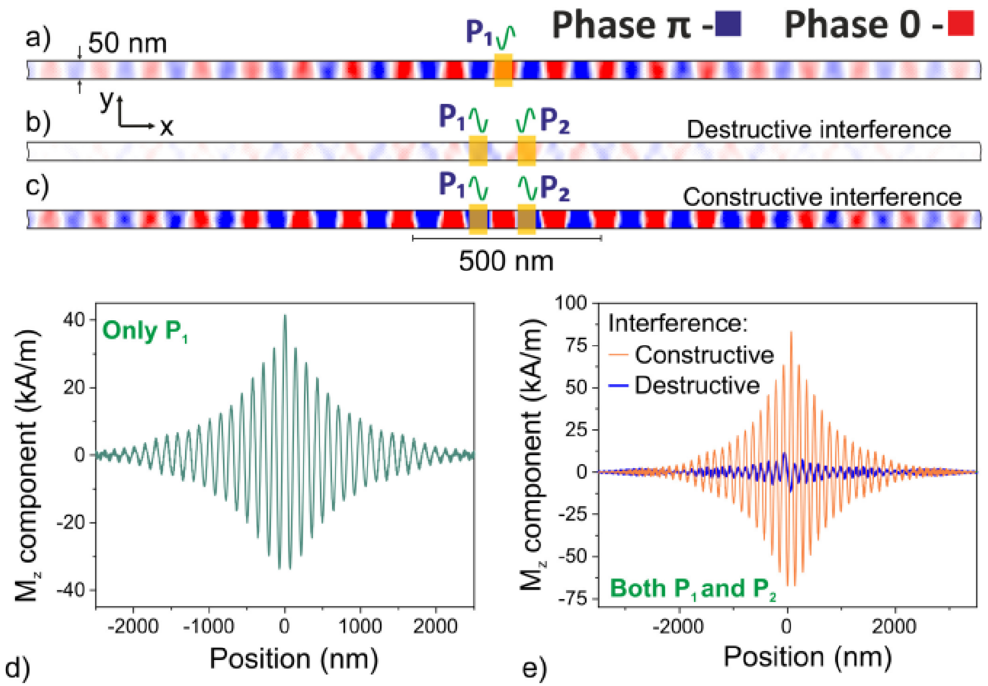
# Micromagnetic Simulations




S. Zhang *Scientific Reports* (**2016**)

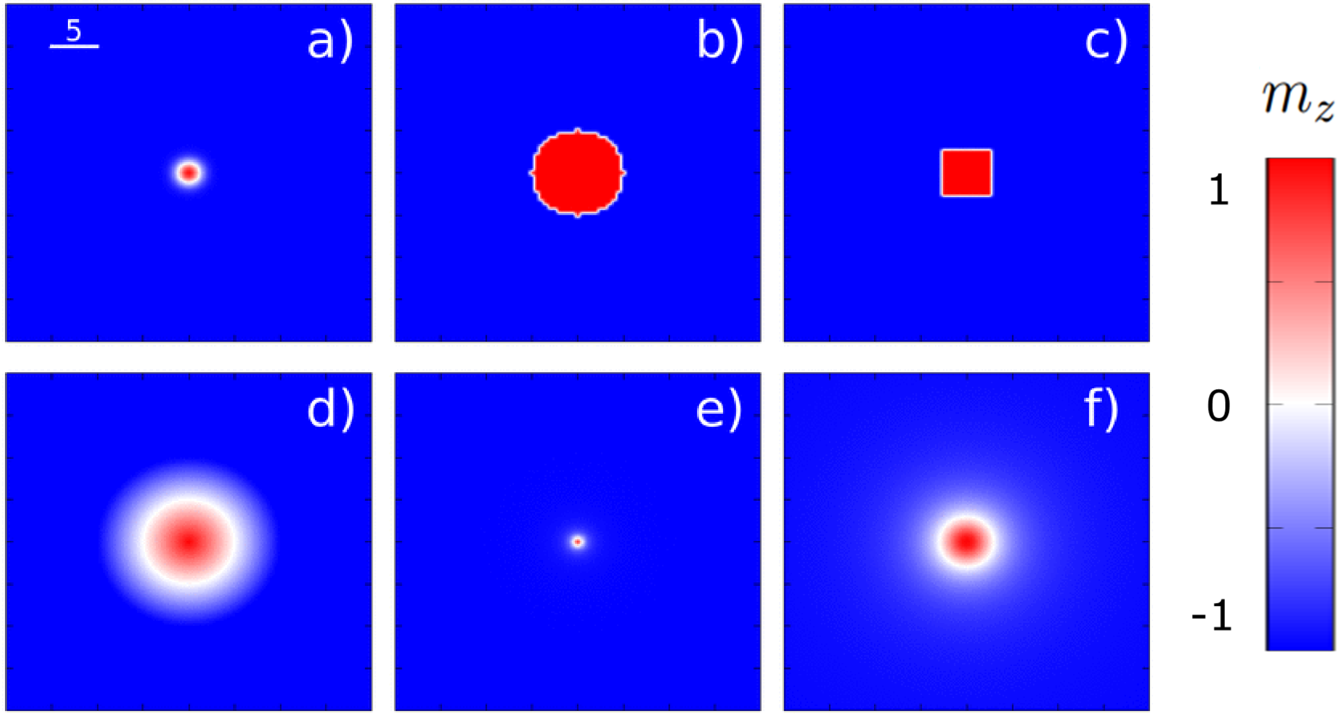



S. Zhang *Scientific Reports* (**2016**)

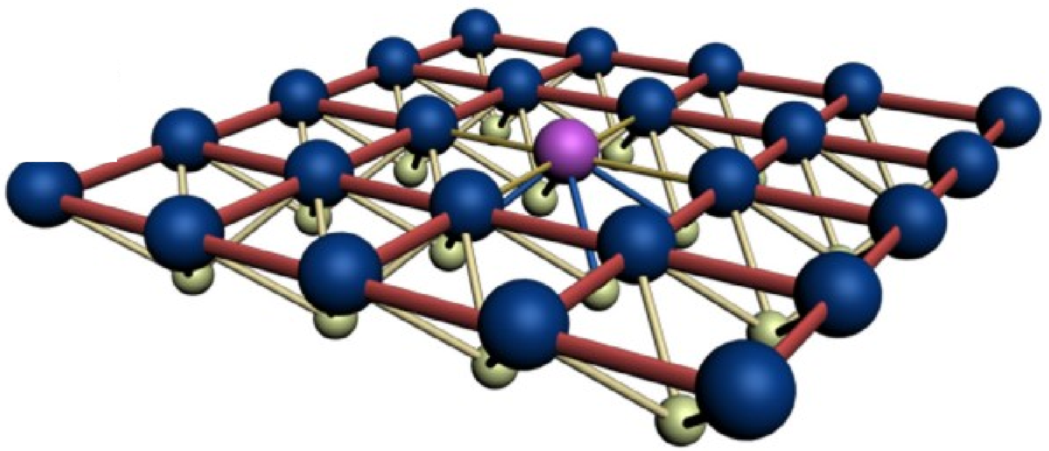
**mumax$^3$**

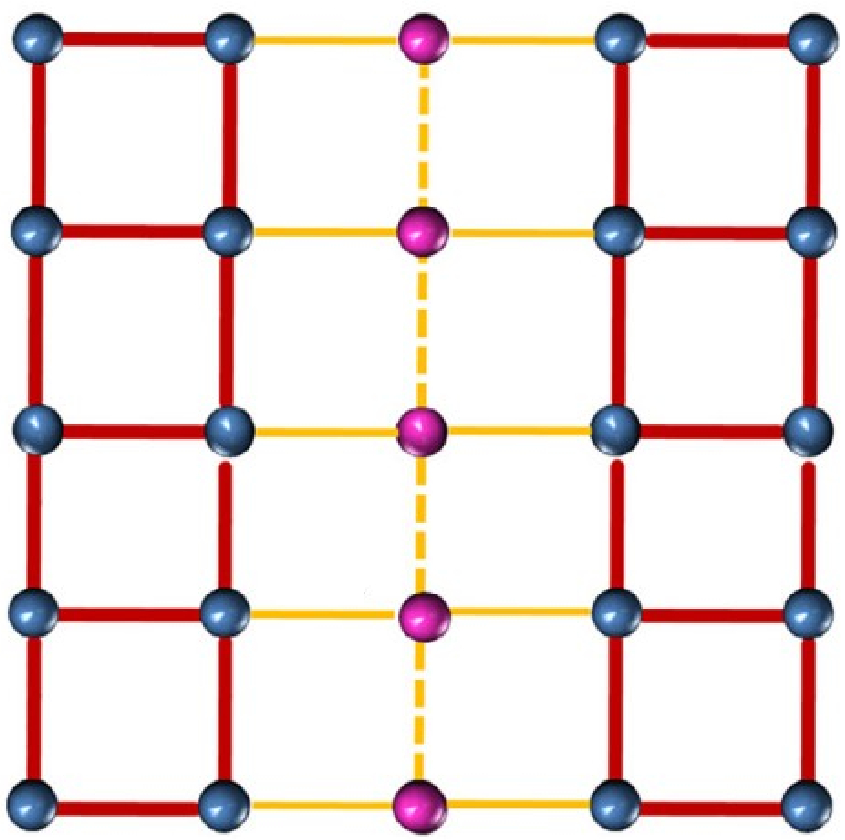
GPU-accelerated micromagnetism

# Micromagnetic simulation



# Linear Defects and skyrmions (LD)





C. Navau *JMMM* (**2018**)

- A defect can be an atomic substition, a vacancy...
- At the micromagnetic scale they can be modeled as local variations of physical parameters

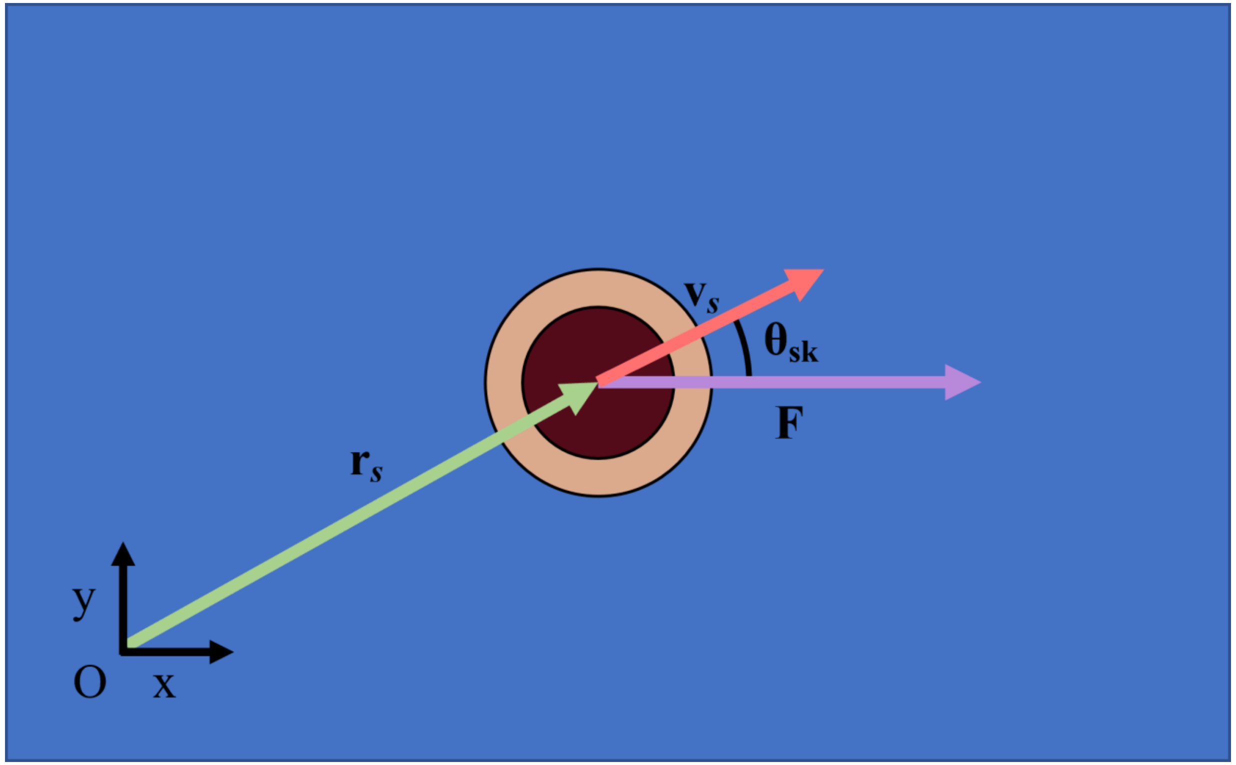
# Rigid Model: Basics

$$(\mathbb{G} - M_s\alpha\mathbb{D})\mathbf{v}_s + \gamma M_s^2\mathbf{F} = 0$$

$$\mathbb{G}_{\nu\upsilon} = \int_V \mathbf{M}_0 \cdot \left(\frac{\partial \mathbf{M}_0}{\partial \nu} \times \frac{\partial \mathbf{M}_0}{\partial \upsilon}\right) \mathrm{d}V'$$

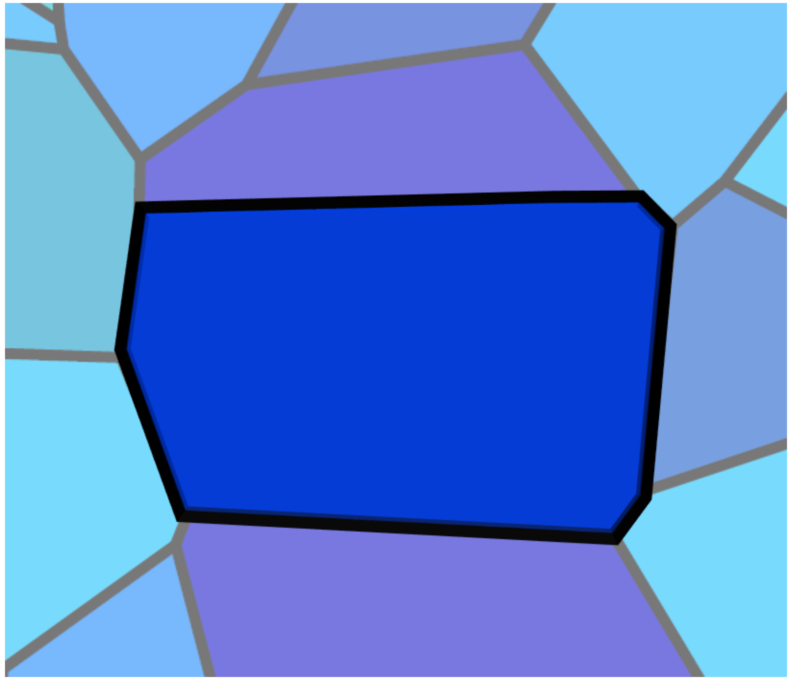
$$\mathbb{D}_{\nu\upsilon} = \int_V \frac{\partial \mathbf{M}_0}{\partial \nu} \cdot \frac{\partial \mathbf{M}_0}{\partial \upsilon} \mathrm{d}V',$$

$$\mathbb{G} = \begin{pmatrix} 0 & G \\ -G & 0 \end{pmatrix} \quad \mathbb{D} = \begin{pmatrix} D & 0 \\ 0 & D \end{pmatrix}$$



# 7. Paper C: Modeling Granularity

$$\mathbf{F}_{\mathrm{ld}} = \frac{K a^2 \gamma}{\mu_0 M_s} \frac{4\,\Delta K\, R^2 r (R^2 - r^2)}{(R^2 + r^2)^3}\, \hat{\mathbf{r}}$$



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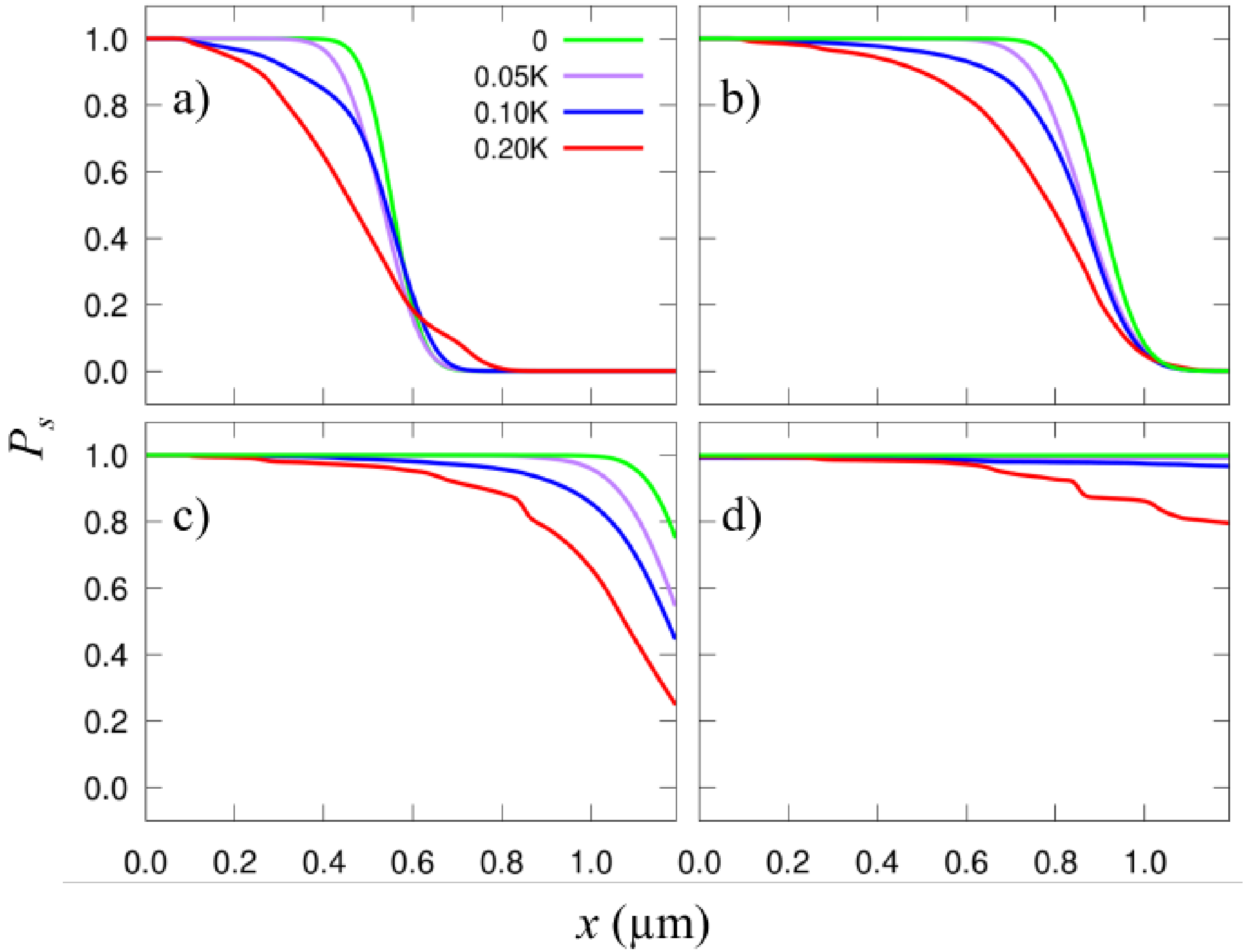
$$\mathbf{F}_{\mathrm{gr}} = f_G \left[ \arctan\left(\frac{L_g/2}{r_0}\right) \left(\frac{-2}{r_0} + \frac{R^2}{r_0^3}\right) + \frac{R^2 L_g}{2 r_0^2 [(L_g/2)^2 + r_0^2]} \right]_{r_-}^{r_+} \hat{\mathbf{r}}$$

# 7. Paper D: Results

- As we increase the intensity of the defects, the transport of the skyrmions is delayed.
- If we further increase it, there is a probability that the skyrmion never reaches the end of the track.



# 7. Paper D: Results

- As we reduce the size of the grains (hence, raising the number of them), the dynamics of the skyrmions are less affected.
- As expected, if the grains are much smaller than the size of the skyrmion, the force field that the skyrmion feels is neglectable