Probabilistic modeling of resistive switching in emerging ReRAM cells

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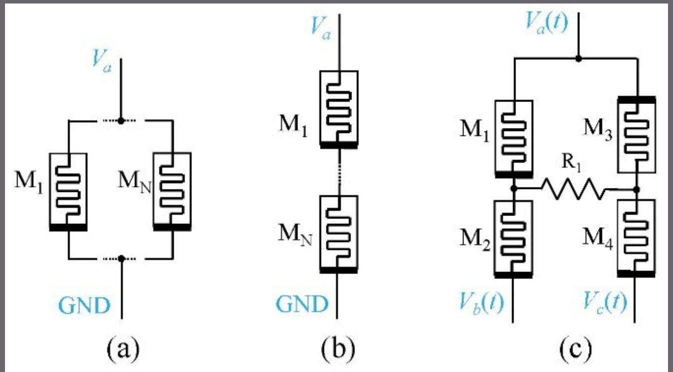
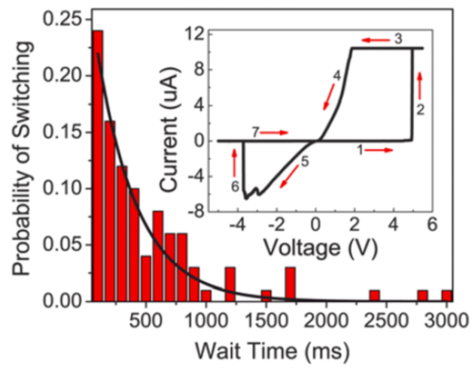
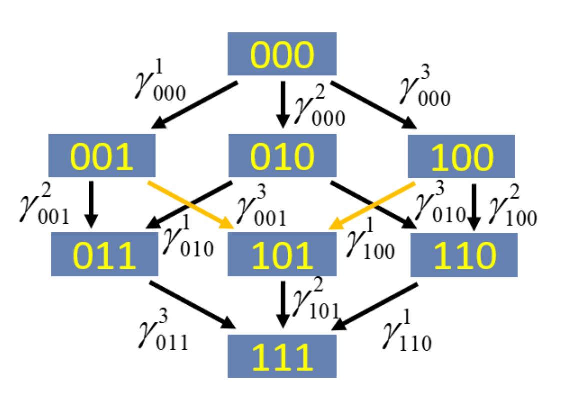
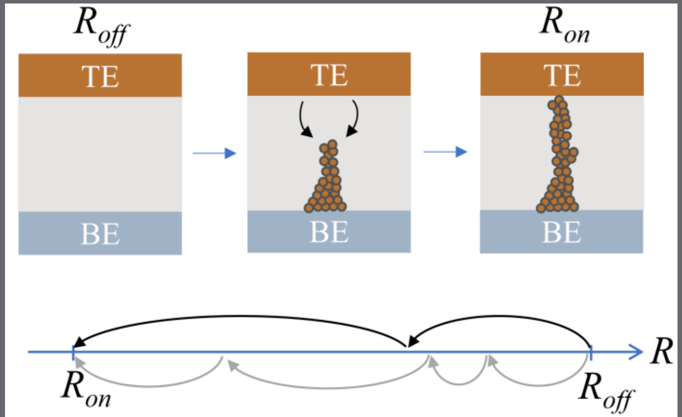






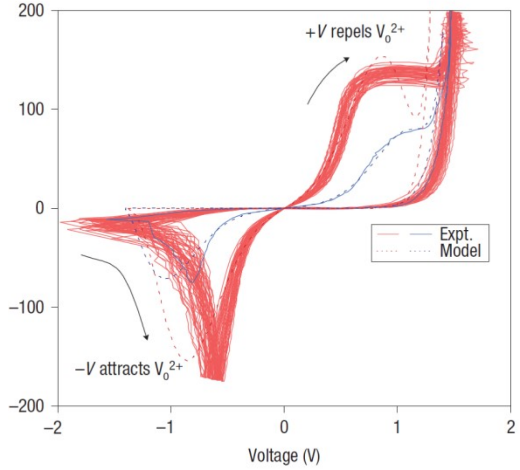


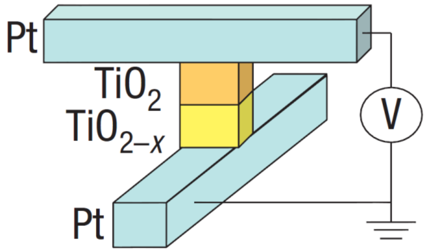
June 16, 2023

IWCN2023, Barcelona

Motivation

OxRAM (VCM)





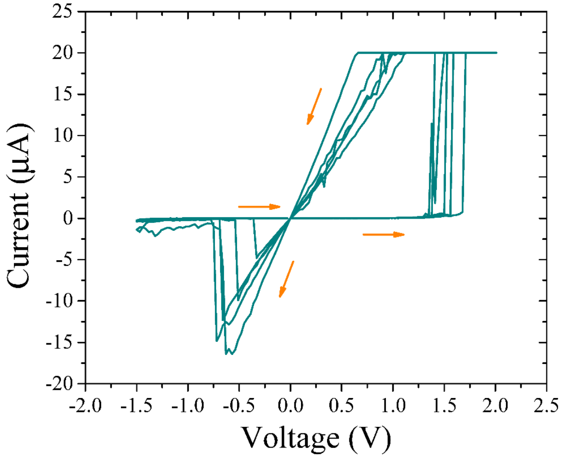
nature Vol 453 | 1 May 2008 | doi:10.1038/nature06932

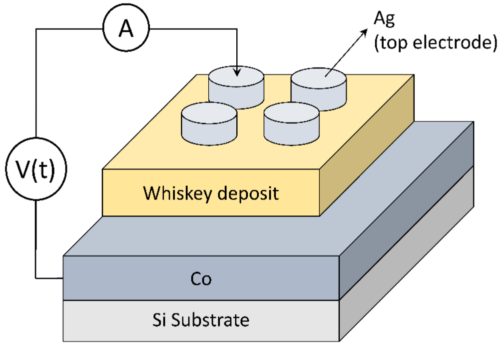
LETTERS

The missing memristor found

Dmitri B. Strukov[1], Gregory S. Snider[1], Duncan R. Stewart[1] & R. Stanley Williams[1]

CBRAM (ECM)





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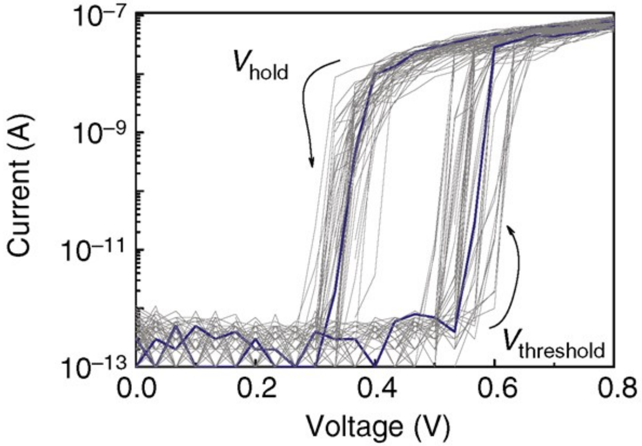
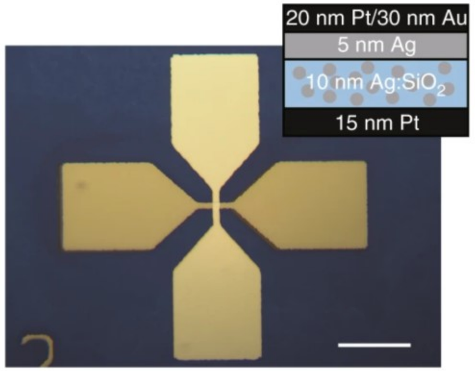
Research Article Open Access

Whisky-Born Memristor

Jinsun Kim, Vincent J. Dowling, Timir Datta, Yuriy V. Pershin

Diffusive memristors





H. Jiang et al., Nature Comm. **8**, 882 (2017)

Memristive models

$$\begin{cases} I = R_M^{-1}(\mathbf{x}, V)V \\ \dfrac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, V) \end{cases}$$

Internal state variable(s)

PROCEEDINGS OF THE IEEE, VOL. 64, NO. 2, FEBRUARY 1976

Memristive Devices and Systems

LEON O. CHUA, FELLOW, IEEE, AND SUNG MO KANG, MEMBER, IEEE

1. Ideal models $V = R_M(q)I \quad R_M(q) = R_0 + \alpha(q - q_0)^2$

2. Phenomenological models (TEAM, VTEAM, etc.)

$$i(t) = \left[R_{ON} + \frac{R_{OFF} - R_{ON}}{w_{off} - w_{on}} \cdot (w - w_{on})\right]^{-1} \cdot v(t)$$

$$\frac{dw(t)}{dt} = \begin{cases} k_{off} \cdot \left(\frac{v(t)}{v_{off}} - 1\right)^{\alpha_{off}} \cdot f_{off}(w), & 0 < v_{off} < v \\ 0, & v_{on} < v < v_{off} \\ k_{on} \cdot \left(\frac{v(t)}{v_{on}} - 1\right)^{\alpha_{on}} \cdot f_{on}(w), & v < v_{on} < 0 \end{cases}$$

S. Kvatinsky, et al., IEEE Trans. Circ. Syst. II 62(8), 786-790 (2015)

3. Physics-based models

$$I = G(x, V_M)V_M$$

$$\dot{x} = A\sinh\left(\frac{V_M}{\sigma_{off}}\right)\exp\left(-\frac{x_{off}^2}{x^2}\right)\exp\left(\frac{1}{1+\beta I\,V_M}\right)H(-V_M) + B\sinh\left(\frac{V_M}{\sigma_{on}}\right)\exp\left(-\frac{x^2}{x_{on}^2}\right)\exp\left(\frac{I\,V_M}{\sigma_p}\right)H(V_M)$$

$$G(x, V_M) = G_M x + a\exp\left(b\sqrt{|V_M|}\right)(1 - x)$$

J. P. Strachan, et al., IEEE Trans. El. Dev. 60, 2194 (2013)

Non-deterministic approaches:

- Monte Carlo simulations
- Stochastic differential equations

$$\{y(t)\}_\xi = \{g(x,u,t)\}_\xi \, u(t),$$
$$\dot{x} = f(x,u,t) + H(x,u,t)\xi(t),$$
$$\langle \xi_i(t) \rangle = 0, \quad \langle \xi_i(t)\xi_j(t') \rangle = k_{ij}(t,t'), \quad i,\ j = 1,\ldots,n,$$

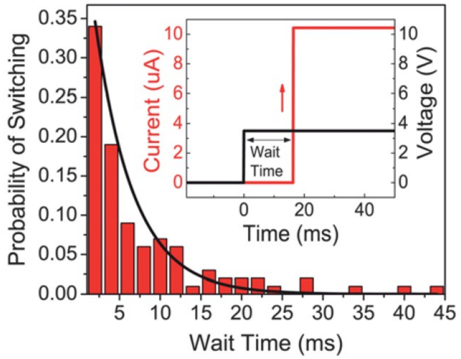
YVP and M. Di Ventra, , Advances in Physics 60, 145-227 (2011)

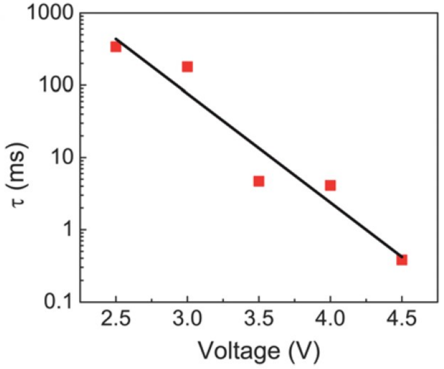
$\langle R(t) \rangle$ $\quad \langle T_{SET} \rangle$ $\quad \left\langle \left(R(t) - \langle R(t) \rangle \right)^2 \right\rangle$ $\quad R(t)$

Computationally-intensive and complex for analytical analysis

Need for a new modeling approach

Distribution of wait time

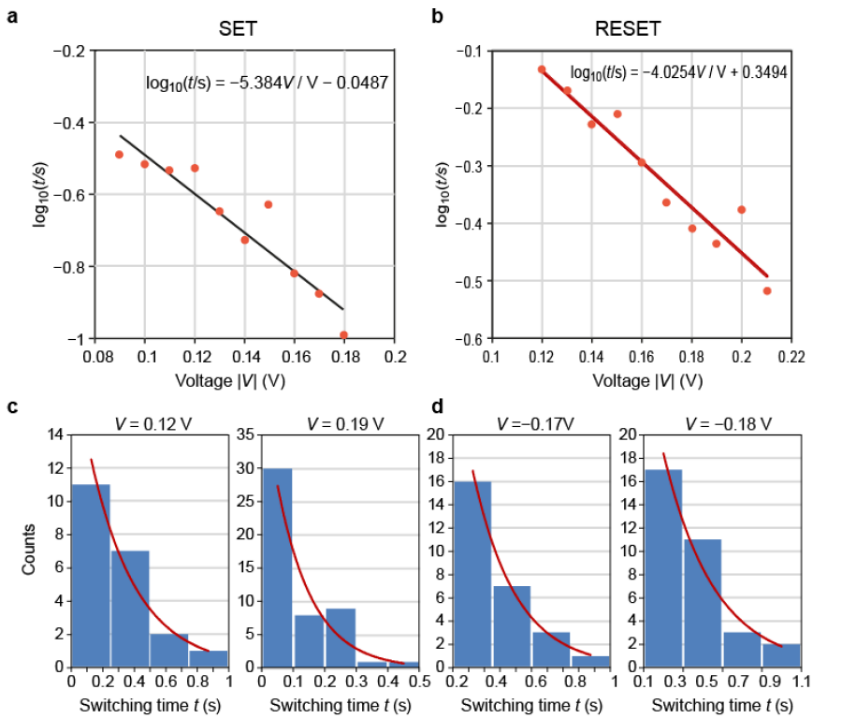


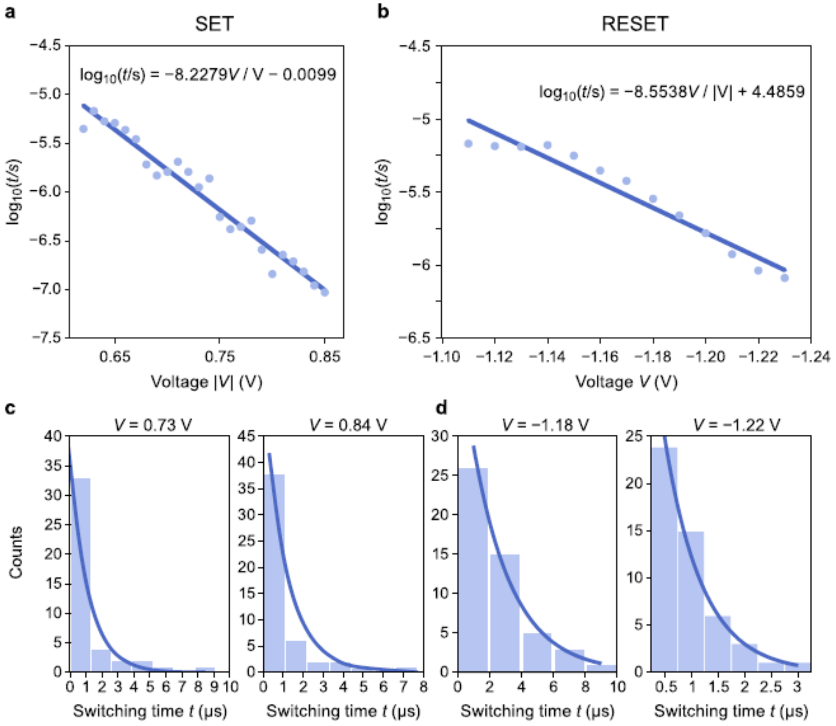


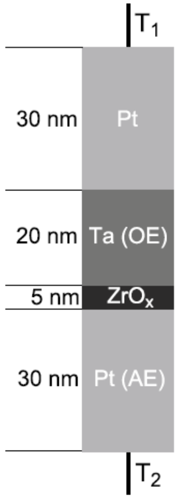
Ag/a-Si/Pt

$$P(t)=\frac{\Delta t}{\tau}e^{-\frac{t}{\tau}}\qquad \tau(V)=\tau_0 e^{-\frac{V}{V_0}}$$

SET and RESET







ECM device (Pt/SiO_2/CuS/Cu/Pt)

VCM device

S. Gaba *et al*, Nanoscale **5**, 5872 (2013)

R. Naous et al., Scientific Reports 11:4218 (2021)

Master equation model

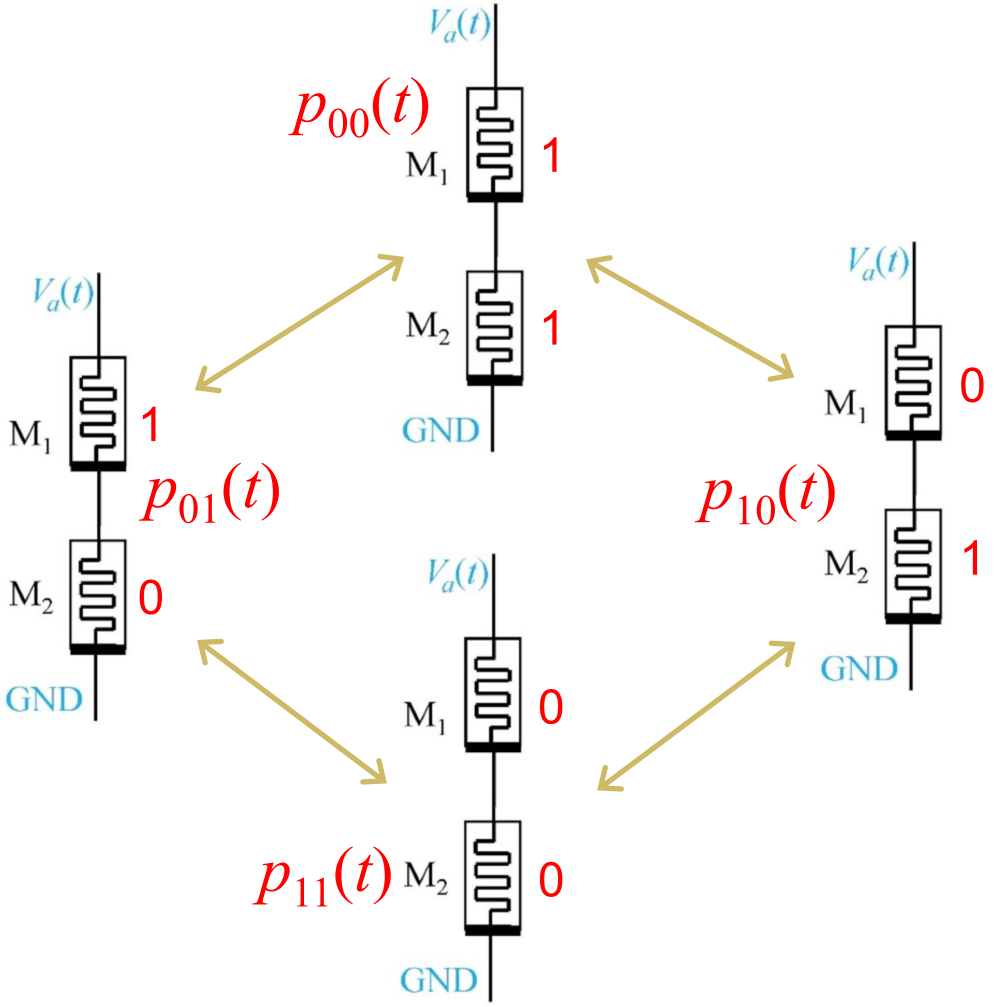
Binary memristors

R_{ON} : 1
R_{OFF} : 0

Transition rates:

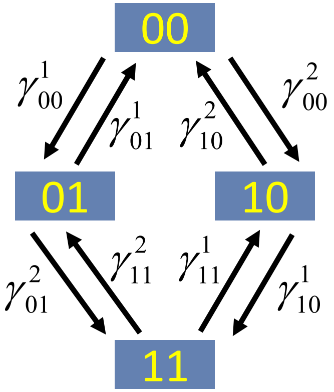
$$\gamma_{0\to1}(V)=\begin{cases}\left(\tau_0 e^{-V/V_0}\right)^{-1} & V>0\\ 0 & \text{otherwise}\end{cases}$$

$$\gamma_{1\to0}(V)=\begin{cases}\left(\tau_1 e^{-V/V_1}\right)^{-1} & V<0\\ 0 & \text{otherwise}\end{cases}$$



$$\frac{dp_\Theta}{dt}=\sum_{m=1}^{N}\left(\gamma^m_{\Theta_m}p_{\Theta_m}(t)-\gamma^m_{\Theta}p_\Theta(t)\right)$$

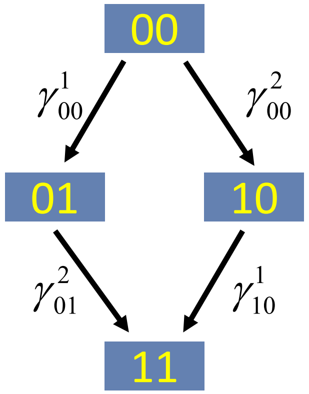
Master equation



V. J. Dowling, V. A. Slipko, Y. V. Pershin, Chaos, Solitons & Fractals **142**, 110385 (2021)

Selected results

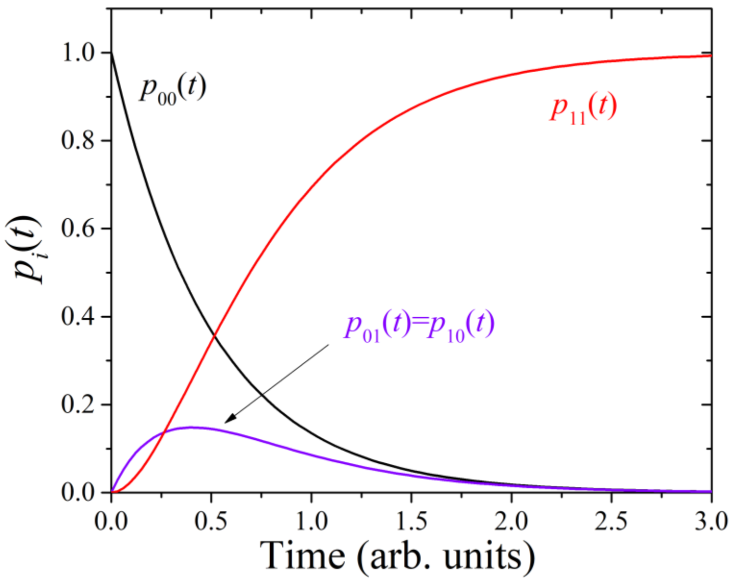
Two identical memristors connected in-series (or in-parallel), $V_a > 0$, $p_{00}(0) = 1$.



$$\frac{\mathrm{d}p_{00}(t)}{\mathrm{d}t} = -2\gamma^1_{00} p_{00}$$

$$\frac{\mathrm{d}p_{01}(t)}{\mathrm{d}t} = \gamma^1_{00} p_{00} - \gamma^2_{01} p_{01}$$

$$\frac{\mathrm{d}p_{11}(t)}{\mathrm{d}t} = 2\gamma^2_{01} p_{01}$$



$$p_{00}(t) = e^{-2\gamma^1_{00} t}$$

$$p_{01}(t) = p_{01}(t) = \frac{\gamma^1_{00}}{\gamma^2_{01} - 2\gamma^1_{00}} \left(e^{-2\gamma^1_{00} t} - e^{-\gamma^2_{01} t} \right)$$

$$p_{11}(t) = 1 - p_{00}(t) - 2p_{01}(t)$$

Average circuit switching time:

$$\langle T_{11} \rangle = \int_0^\infty t\, 2\gamma^2_{01} p_{01}(t) \mathrm{d}t = \frac{1}{2\gamma^1_{00}} + \frac{1}{\gamma^2_{01}}$$

Average resistance of memristor 1:

$$\langle R_1 \rangle(t) = R_{OFF} \left(p_{00}(t) + p_{10}(t) \right) + R_{ON} \left(p_{01}(t) + p_{11}(t) \right)$$

More complex cases

N **identical memristors connected in-series (or in-parallel)**, $V_a > 0$, $p_{0,,,0}(0)=1$.

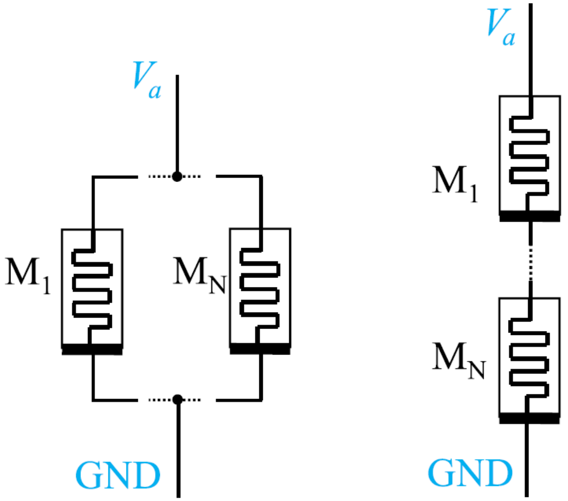
$$\frac{\mathrm{d}p_0(t)}{\mathrm{d}t} = -N\gamma_0 p_0$$

$$\frac{\mathrm{d}p_m(t)}{\mathrm{d}t} = m\gamma_{m-1}p_{m-1} - (N-m)\gamma_m p_m$$

$$\sum_{m=0}^{N}\binom{N}{m} p_m(t) = 1$$

$$p_m(t) = \sum_{i=0}^{m}\left(\prod_{k=1}^{m} b_k\right)\left(\prod_{j=0,j\neq i}^{m}\frac{1}{a_j - a_i}\right)e^{-a_i t}$$

$$a_m = (N-m)\gamma_m$$
$$b_m = m\,\gamma_{m-1}$$



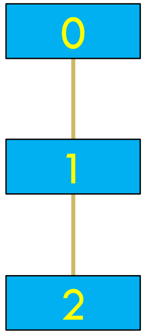
Network switching time: $$\langle T_N \rangle = \sum_{j=0}^{N-1}\frac{1}{(N-j)\gamma_j}$$

In-parallel connected memristors: $$\langle T_{\|,N} \rangle = \frac{1}{\gamma_0^1}\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{N}\right)$$

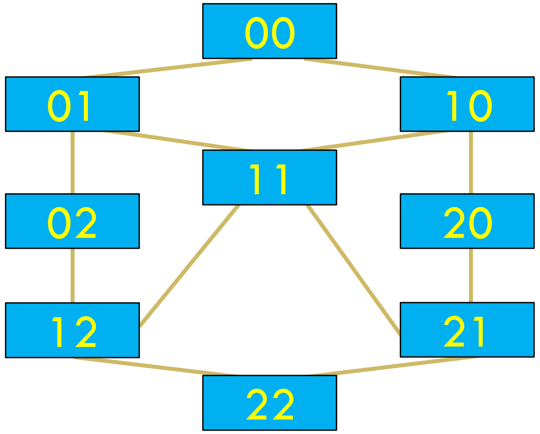
V. J. Dowling, V. A. Slipko, Y. V. Pershin, Chaos, Solitons & Fractals **142**, 110385 (2021)

Multi-state memristors

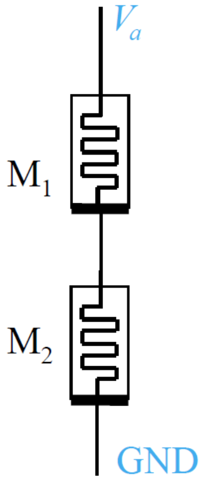
Examples of transition schemes:

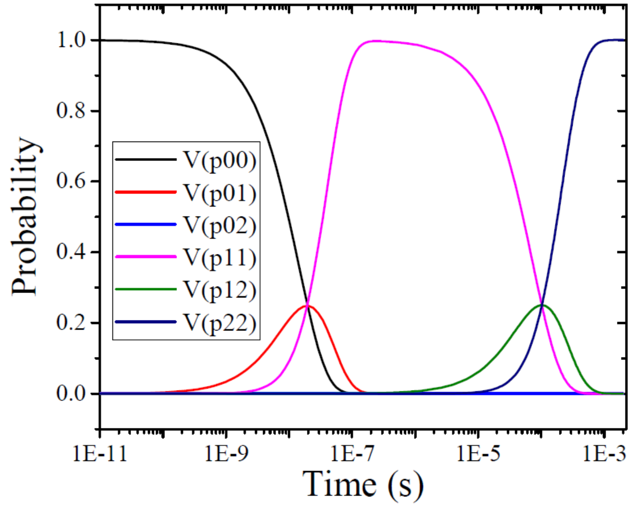


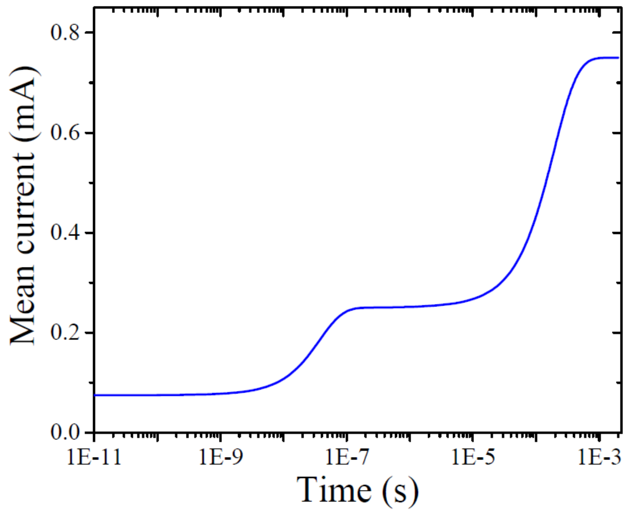
3-state memristor



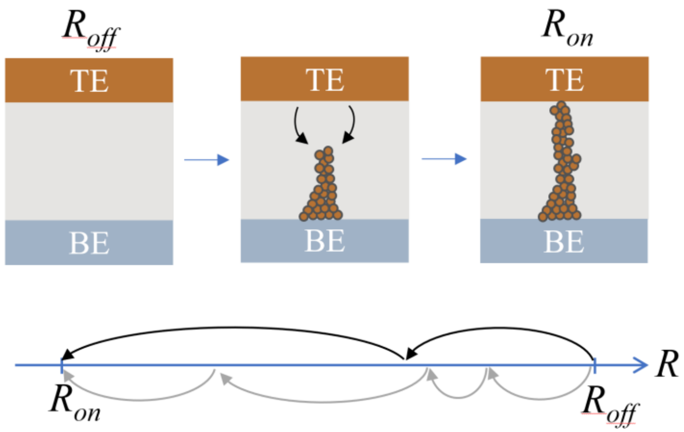
Two 3-state memristors

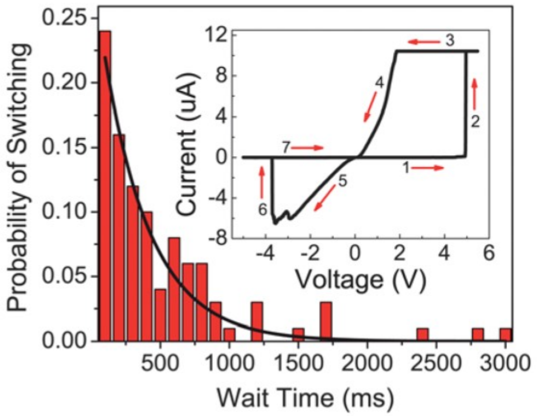




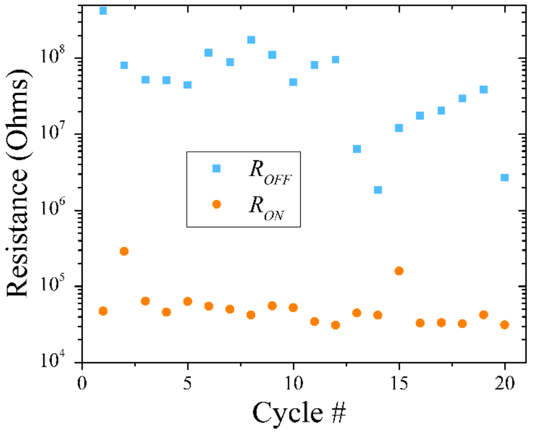


Continuous model





S. Gaba *et al*, Nanoscale **5**, 5872 (2013)



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Whisky-Born Memristor

Jinsun Kim, Vincent J. Dowling, Timir Datta, Yuriy V. Pershin

$p(x,t)$: state probability distribution function

$$\langle I \rangle = \left\langle \frac{V}{R(x,V)} \right\rangle \equiv \overline{R}^{-1}(V)V$$

$$\frac{\partial p(x,t)}{\partial t} = \int_a^b \gamma(x',x,V(x'))p(x',t)dx' - p(x,t)\int_a^b \gamma(x,x',V(x))dx'$$

incoming outgoing

$$x \equiv R \qquad R \in [R_{\rm on}, R_{\rm off}]$$

uniform distribution of jumps

$$\gamma(R',R,V) = \begin{cases} \alpha_{10} e^{\frac{|V|}{V_{10}}}, & V>0,\ R'<R \\ \alpha_{01} e^{\frac{|V|}{V_{01}}}, & V<0,\ R'>R \\ 0, & \text{otherwise} \end{cases}$$

exponential distribution of jumps

$$\gamma(R,R',V) = \begin{cases} \alpha_{10} e^{\frac{|V|}{V_{10}} - \frac{|R-R'|}{R_0}}, & V>0,\ R<R' \\ \alpha_{01} e^{\frac{|V|}{V_{01}} - \frac{|R-R'|}{R_0}}, & V<0,\ R>R' \\ 0, & \text{otherwise} \end{cases}$$

V. A. Slipko, Y. V. Pershin, Phys. Rev. E (in press); arXiv:2302.03079 (2023)

Uniform distribution of jumps

Response to a steplike voltage

Laplace transform: $\tilde{r}(R,p) = \int_0^{\infty} r(R,t)\exp(-pt)\mathrm{d}t$

$$[p+\gamma(R_{off}-R)]\,\tilde{r}(R,p) = \gamma \int_{R_{on}}^{R} \tilde{r}(R',p)\mathrm{d}R' + r(R,0)$$

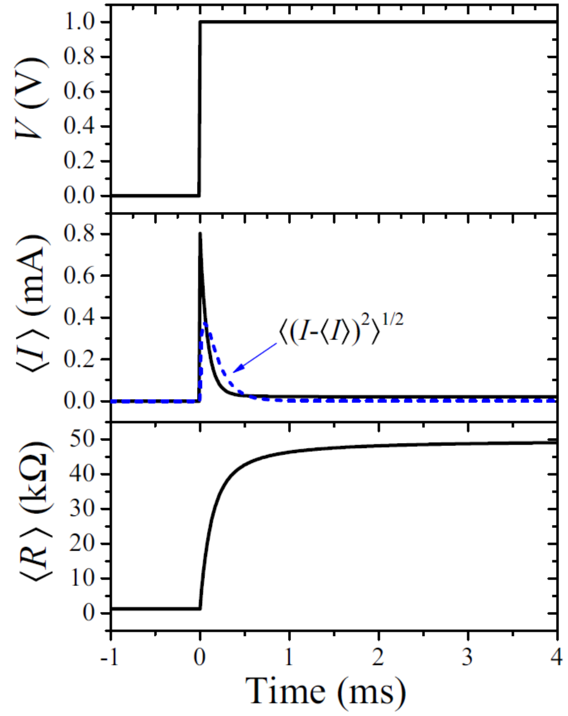
$$\tilde{r}(R,p) = \frac{r(R,0)}{[p+\gamma(R_{off}-R)]} + \frac{C(p)+\gamma\int_{R_{on}}^{R} r(R',0)\mathrm{d}R'}{[p+\gamma(R_{off}-R)]^2}$$

Inverse Laplace transform:

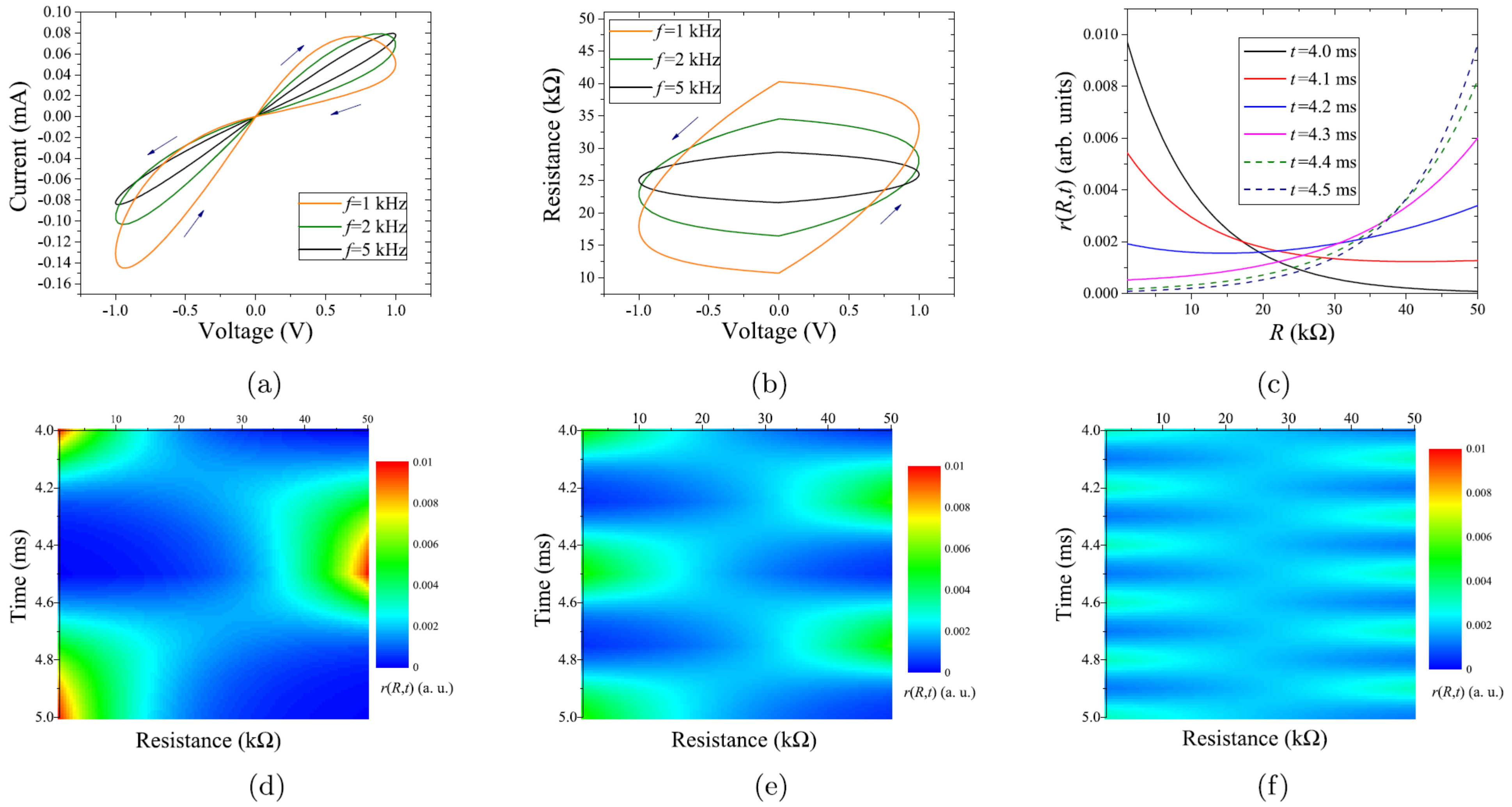
$$r(R,t) = \gamma t e^{-\gamma(R_{off}-R)t}\int_{R_{on}}^{R} r(R',0)\mathrm{d}R' + r(R,0)e^{-\gamma(R_{off}-R)t}$$

$$\langle R\rangle(t) = \int_{R_{on}}^{R_{off}} r(R,t)R\mathrm{d}R = R_{off} - \frac{1-e^{-\gamma_{10}(R_{off}-R_{on})t}}{\gamma_{10}t}$$

$$\langle (R-\langle R\rangle)^2\rangle = \frac{1-2\gamma_{10}t(R_{off}-R_{on})e^{-\gamma_{10}(R_{off}-R_{on})t} - e^{-2\gamma_{10}(R_{off}-R_{on})t}}{(\gamma_{10}t)^2}$$



Response to a sinusoidal voltage

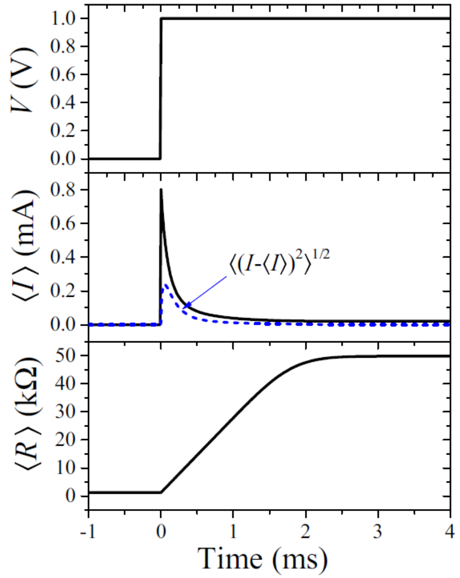


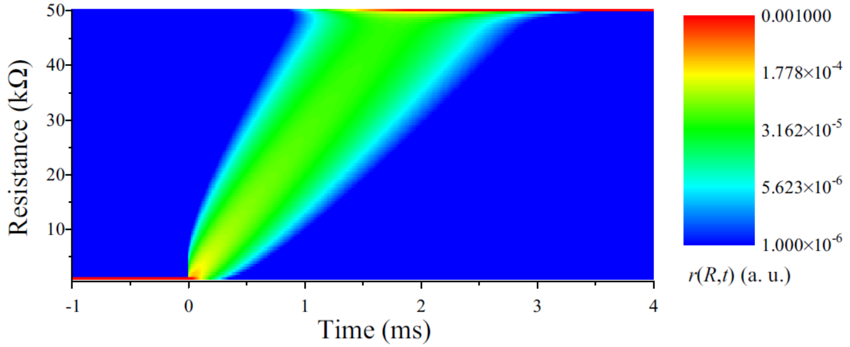
Exponential distribution of jumps

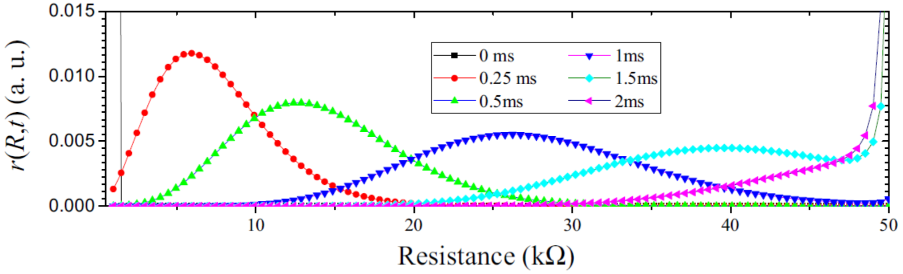
Response to a steplike voltage

$$r(R,t) = r(R,0)e^{-\Gamma(R)t} + e^{-\Gamma(R)t}\int_{R_{on}}^{R} \frac{\mathrm{d}R_1}{R_0} r(R_1,0)\exp\left\{-\frac{R-R_1}{R_0}\right\} I(t,R,R_1),$$

$$I(t,R,R_1) = \frac{1}{2\pi i}\oint_{|\zeta|=1}\frac{\mathrm{d}\zeta}{1+[\delta-\delta_1]\zeta}\exp\left\{\frac{\gamma_{10}R_0 t}{\zeta} + \frac{\zeta\left(\ln\left[1+(\delta-\delta_1)\zeta\right] + \frac{R-R_1}{R_0}\right)}{1+\zeta\delta}\right\}$$







$$r(R,0) = \delta(R - R_{on})$$

Short-time behavior:

$$\langle R\rangle = R_{on} + R_0^2\gamma_{10}t,$$

$$\langle (R-\langle R\rangle)^2\rangle = 2R_0^3\gamma_{10}t$$

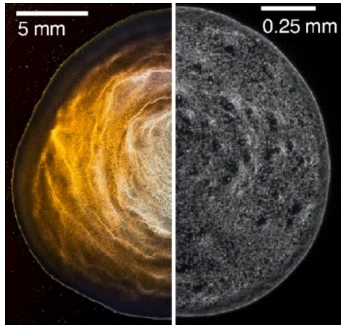
V. A. Slipko, Y. V. Pershin, Phys. Rev. E (in press); arXiv:2302.03079 (2023)

Conclusion

- Novel modeling approaches are needed for simulating circuits with stochastic components
- Certain circuits with stochastic binary/multistate memristors can be efficiently modeled using master equation combined with Kirchhoff's laws
- Analytical solutions in several interesting cases have been found
- An extension to the case of random jumps in the continuous space has been developed

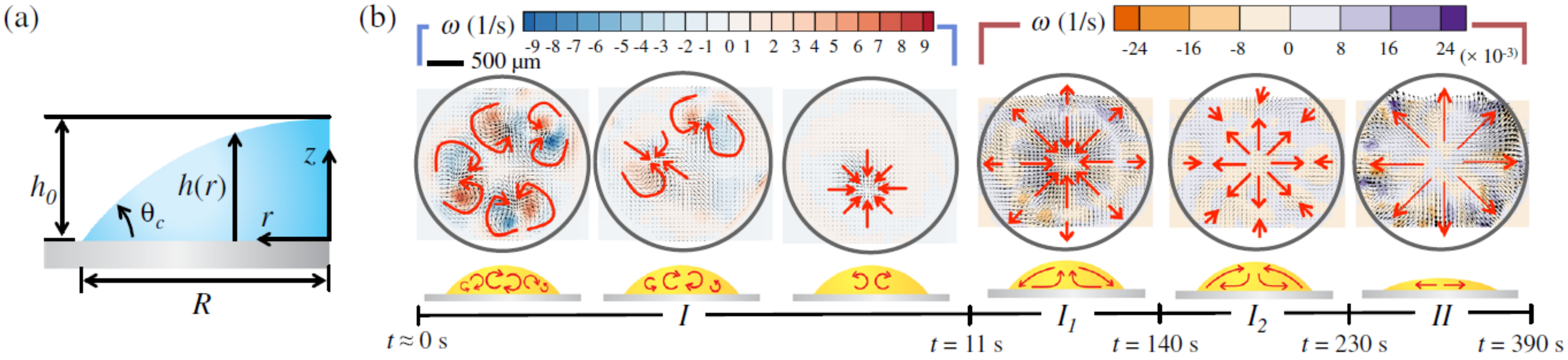
Acknowledgements: NSF SmartState® SC Centers of Economic Excellence (CENPhys)

Uniform coating with whisky deposits



Left: Macallan, UK
Right: Glenlivet

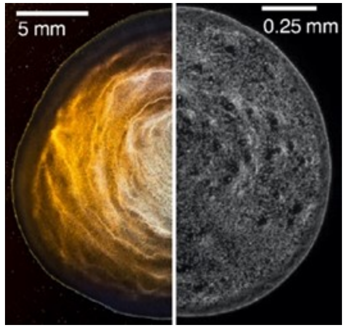
Mechanism: nonuniform particle distributions are prevented by Marangoni flows



Controlled Uniform Coating from the Interplay of Marangoni Flows and Surface-Adsorbed Macromolecules
H. Kim et al., Phys. Rev. Lett. **116**, 124501 (2016)

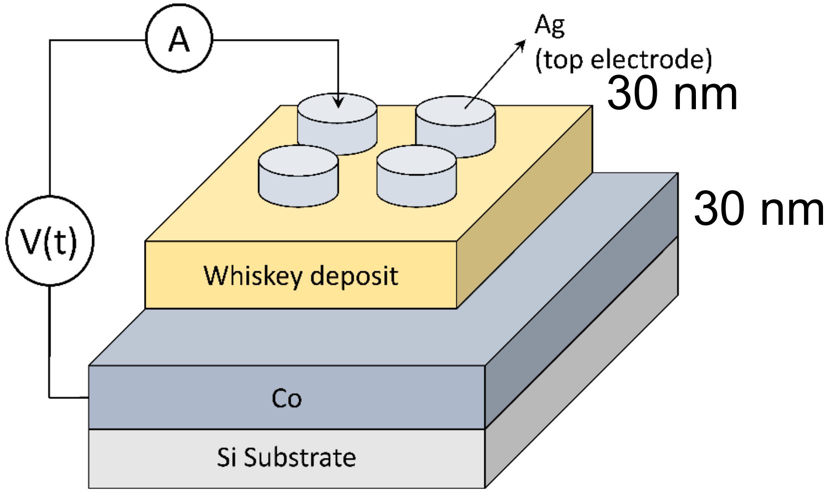
- Has significant cultural and economical value
- The earliest record: 1 June 1494
- Leaves uniform deposits when dries

Coffee ring

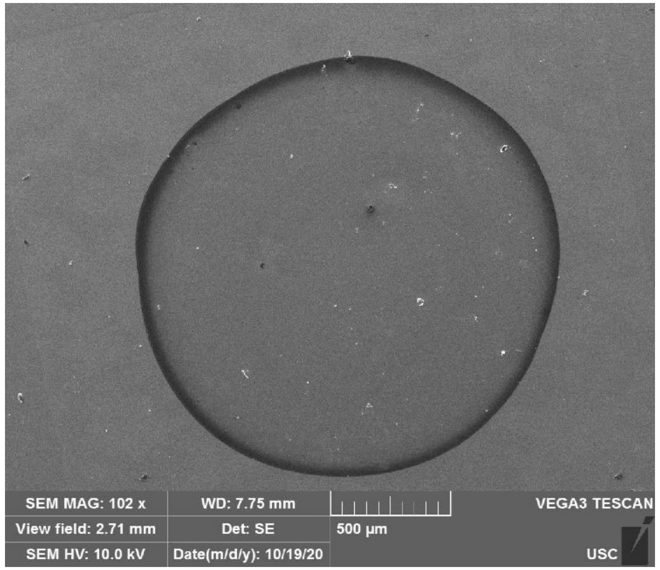


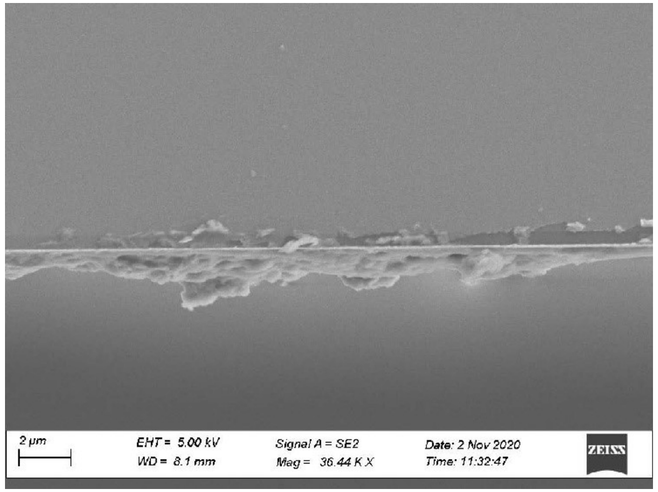
Whisky deposit

Holy memristor

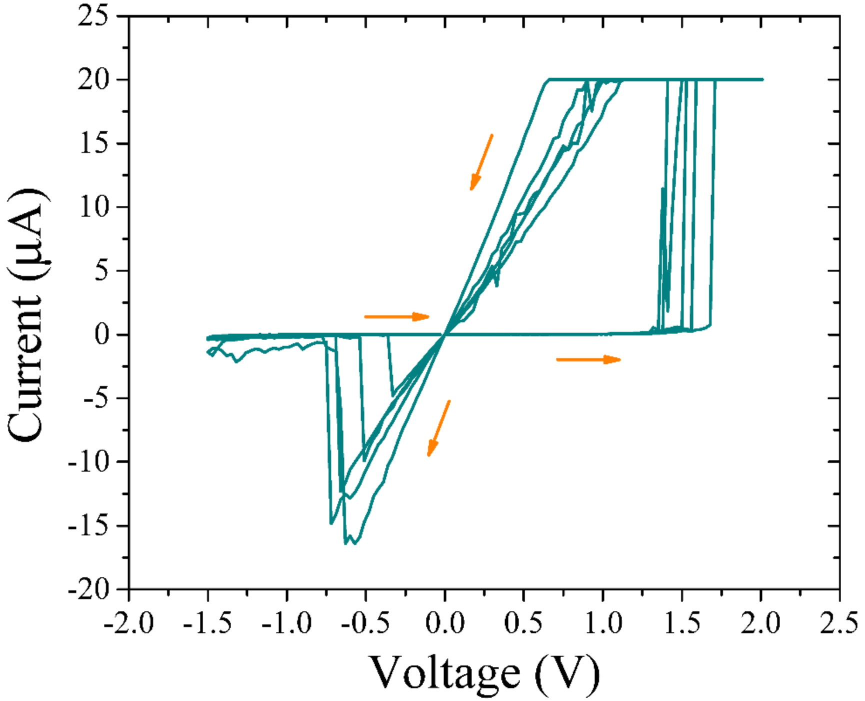


Questa e la vera acqua santa
(this is the real holy water),
Pope Francis, October 9, 2021

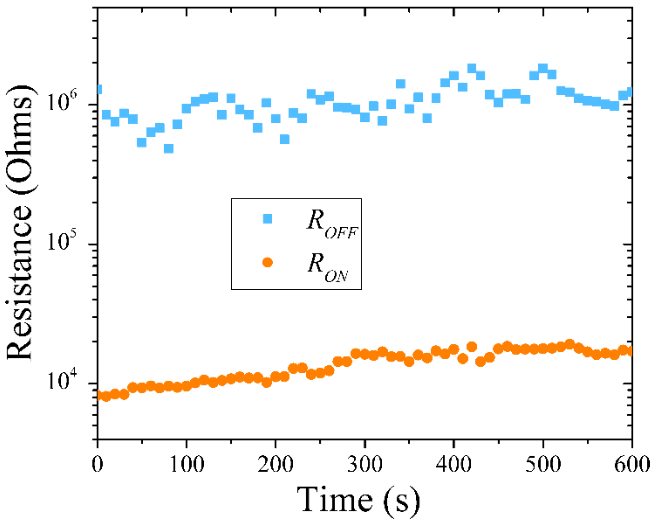




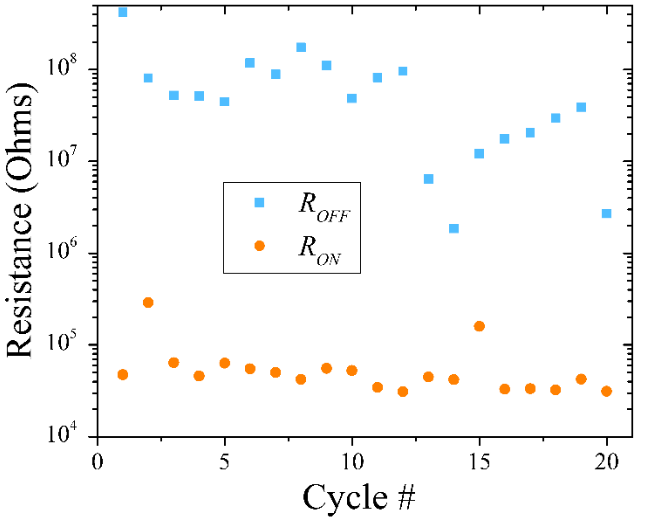
Results



Current (μA)
Voltage (V)
25
20
15
10
5
0
-5
-10
-15
-20
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5



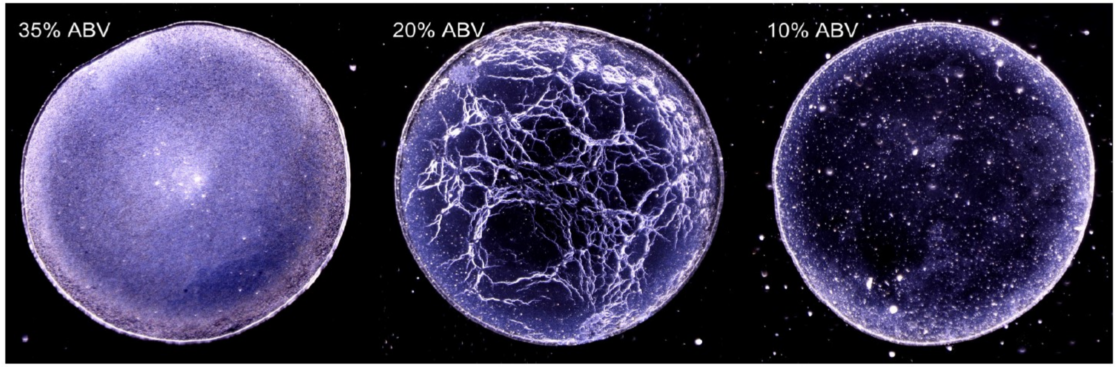
Resistance (Ohms)
Time (s)
10^6
10^5
10^4
0
100
200
300
400
500
600
R_{OFF}
R_{ON}



Resistance (Ohms)
Cycle #
10^8
10^7
10^6
10^5
10^4
0
5
10
15
20
R_{OFF}
R_{ON}

Dilution effects

Rabbit Hole™ bourbon whiskey



uniform deposit weblike structure coffee-ring pattern

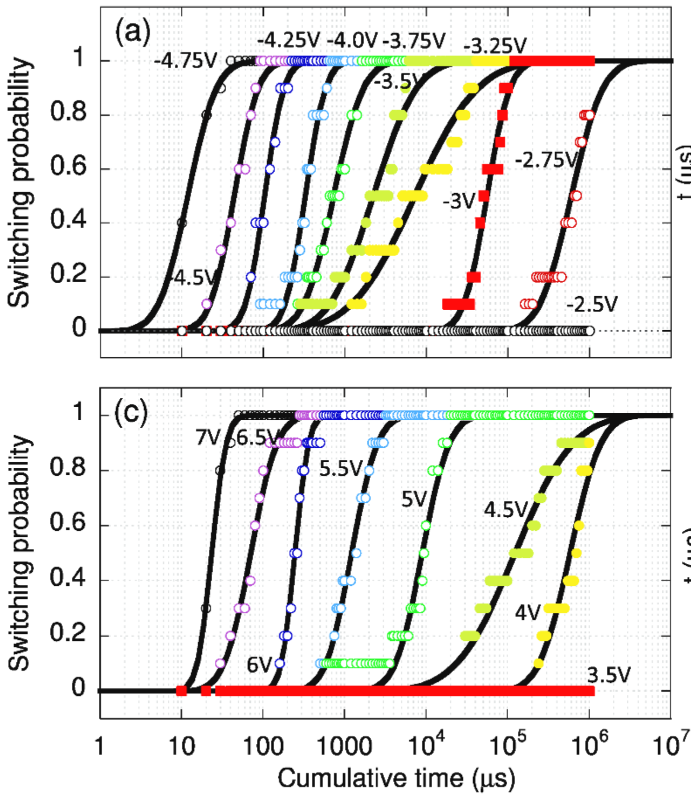
S. J. Williams, et al., Phys. Rev. Fluids **4**, 100511 (2019)

Potential application: counterfeit sensor

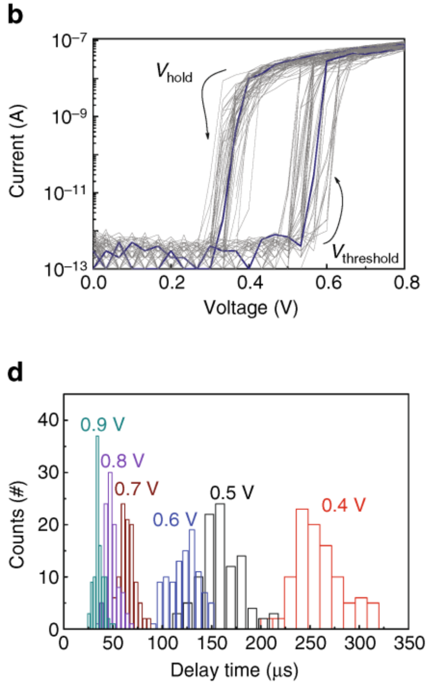
Dilution	1:0	1:1	1:2
Threshold voltage	1.52 V	1.31 V	1.17 V

Kim, Jinsun, et al. "Holy memristor." arXiv preprint arXiv:2111.11557 (2021)

Lognormal switching times



Pt/TiO_2/Pt



Au/Ag/Ag:SiO_2/Pt

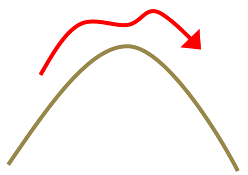
H. Jiang et al., Nature Comm. **8**, 882 (2017)

$$F\left(t_{switch};\tau,\sigma_t\right)=\int_{-\infty}^{t_{switch}}\frac{1}{\sqrt{2\pi}t\sigma_t}\exp[-\frac{(\ln t/\tau)}{2\sigma_t^2}]dt$$

G. Medeiros-Ribeiro *et al*, Nanotechnology **22,** 095702 (2011)

Physical mechanisms

"Poisson" distribution

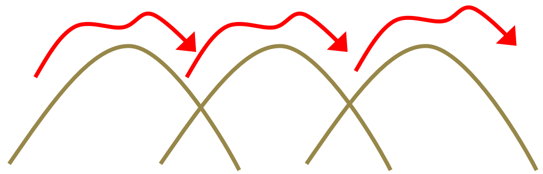


Binary model

Thermal activation over dominant energy barrier

S. Gaba *et al*, Nanoscale **5**, 5872 (2013)

Lognormal distribution



Multi-state model (?)

Thermal activation over several energy barriers (?)

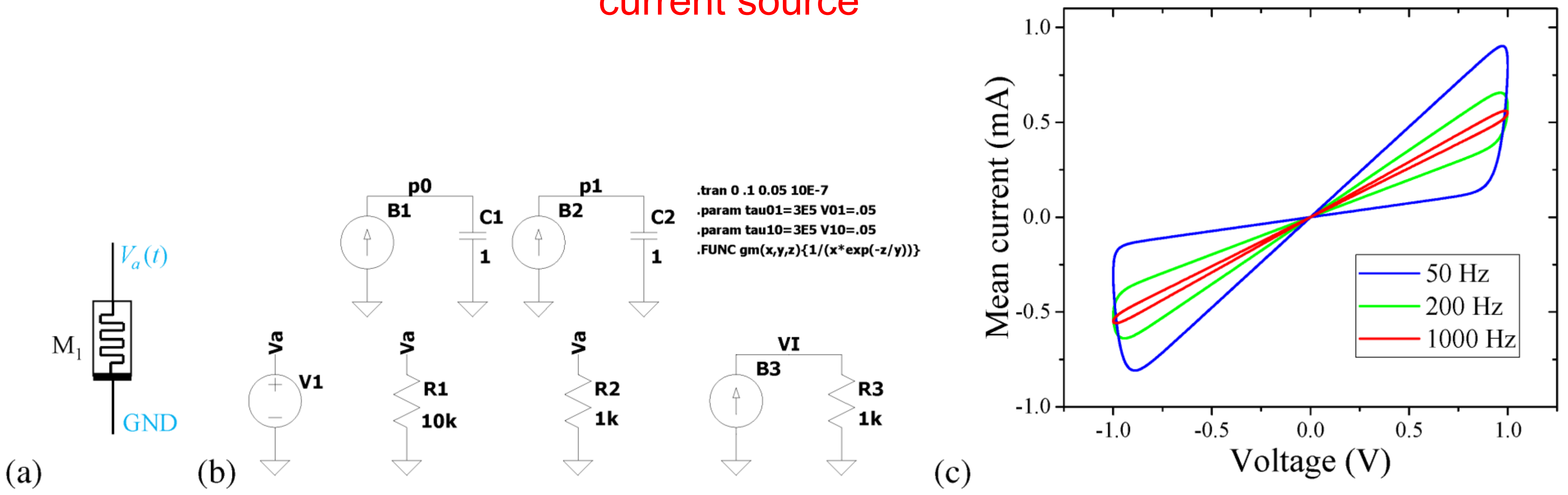
SPICE modeling

Binary memristors

Voltage on capacitor

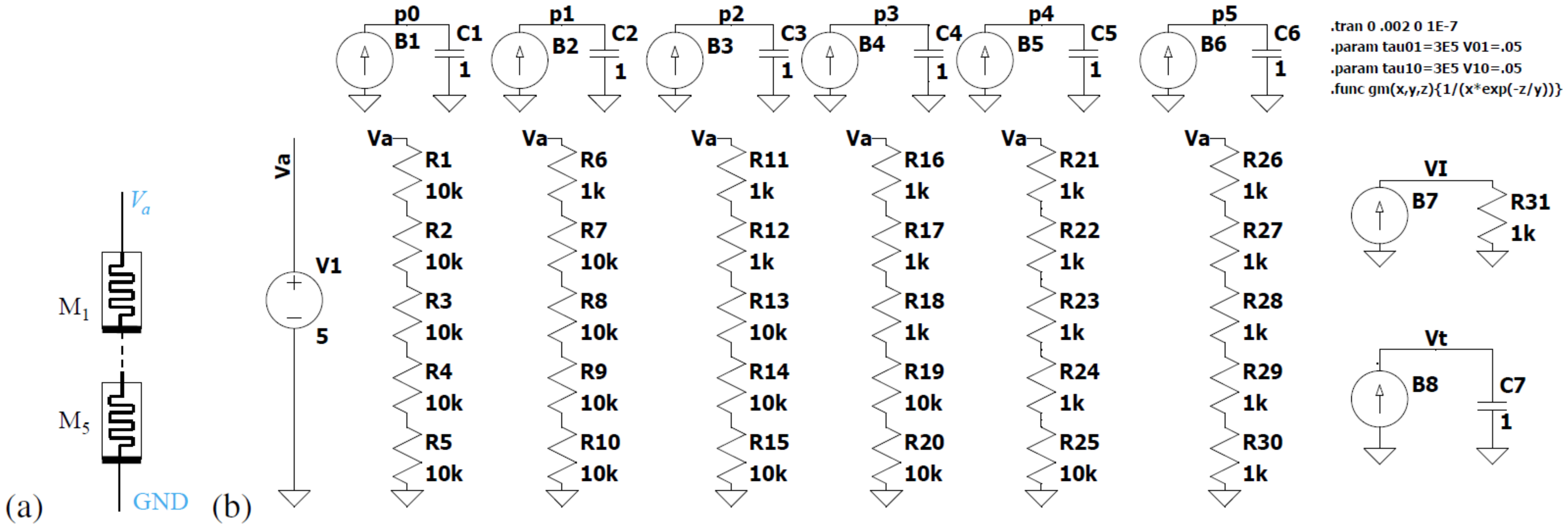
$$\frac{\mathrm{d}p_{\Theta}}{\mathrm{d}t} = \sum_{m=1}^{N}\left(\gamma^{m}_{\Theta_m} p_{\Theta_m}(t) - \gamma^{m}_{\Theta} p_{\Theta}(t)\right)$$

current source



V. J. Dowling, V. A. Slipko, Y. V. Pershin, Radioengineering **30**, 157 (2021)

Five in-series connected memristors



Schematics of SPICE model