General Maxwell-Bloch modelling of self-induced transparency in N-level atom

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Introduction

We investigate self-induced transparency (SIT) effect caused by Rabi oscillations in N-level atom systems implemented in multiple quantum well superlattice. The common models typically implement a two-level Maxwell-Bloch model [1] and consider SIT effect in quantum systems that ideally match with theoretical predictions. The realistic quantum system, as a heterostructure superlattice, however have design-dependent parameters and a ladder of quantum states which cannot be justified by only two levels. We develop a model in which we couple full Maxwell equations to Bloch equations comprised of N levels. We consider scattering of electrons with longitudinal optical phonons, acoustic phonons, interface roughness, impurities and other electrons that provide a realistic disipater within Bloch equations which is similar to the modelling of transport in terahertz quantum cascade lasers [2]. Our model allows propagation of arbitrary input signal and offers detail information of dynamics within N-level atom. This model opens new insights in developing strategies for passive mode locking of THz QCLs via SIT effect.

We analyse ideal two level atom (Fig. 1 and 2) and an exemplary superlattice (Fig. 3) with calculated dephasing time of 0.6 ps, and driven by 3 ps optical pulse at 2 THz frequency (Fig. 4). The system generates partial Rabi flops between the levels at the pump's frequency and complex multiphoton absorption (Fig. 5) that can only be modelled by N–level model.

Model

The general formulation of Maxwell-Bloch model for N-level atom consists of two Maxwell equations:

$$\begin{aligned}\frac{\partial H}{\partial t} &= \frac{1}{\mu}\frac{\partial E}{\partial z}\\ \frac{\partial E}{\partial t} &= \frac{1}{\epsilon}\frac{\partial H}{\partial z} - \frac{\sigma}{\epsilon}E - \frac{1}{\epsilon}\frac{\partial P}{\partial t}\end{aligned} \tag{1}$$

where the polarisation term is calculated by coupling this equation to the minimal set of $N(N+1)/2$ Bloch equations:

$$\begin{aligned}\frac{\partial P}{\partial t} &= 2en_{3D}\sum_j\sum_{i>j}\left(\frac{z_{ji}}{\tau_{||ij}}X_{ij} - z_{ji}\omega_{ij}Y_{ij}\right)\\ \frac{\partial \rho_{ii}}{\partial t} &= -\frac{\rho_{ii}}{\tau_i} + \sum_{j\neq i}\left(\frac{2eE}{\hbar}z_{ij}Y_{ji}\mathrm{sgn}(j-i) + \frac{\rho_{jj}}{\tau_{ji}}\right)\\ &\text{Write only for } i>j \text{ on the left hand side:}\\ \frac{\partial X_{ij}}{\partial t} &= \omega_{ij}Y_{ij} - \frac{X_{ij}}{\tau_{||ij}}\\ &+ \frac{eE}{\hbar}\sum_{k\neq i,j}\left(z_{ik}Y_{kj}\mathrm{sgn}(k-j) - z_{jk}Y_{ik}\mathrm{sgn}(i-k)\right)\\ \frac{\partial Y_{ij}}{\partial t} &= -\omega_{ij}X_{ij} - \frac{Y_{ij}}{\tau_{||ij}} - \frac{eE}{\hbar}\sum_{k\neq i,j}\left(z_{ik}X_{kj} - z_{jk}X_{ik}\right)\\ &- \frac{ez_{ij}E}{\hbar}\left(\rho_{jj} - \rho_{ii}\right)\end{aligned} \tag{2}$$

where ρ_{ii} (populations) are diagonal elements of the density matrix, while the off-diagonal elements are written as $\rho_{ij} = X_{ij} + jY_{ij}$ allowing a system of equations in real plane, τ_i represent the state lifetimes, τ_{ij} are scattering times while $\tau_{||ij}$ are dephasing times modelled as in [2], z_{ij} are dipole elements and n_{3D} is volume doping density. The Maxwell equations are solved on a Yee grid in order to ensure numerical stability of the system. The normalisation condition $\sum_i \rho_{ii} = 1$ is implemented by replacing Bloch equation for the last state as $\rho_{NN} = 1 - \sum_{i\neq N}\rho_{ii}$ and in order to increase the numerical accuracy of the system, Bloch equations are solved within predictor-corrector algorithm. For $N = 2$, this system directly folds into two level atom model [1]. Quantum system parameters $(z_{ij}, \tau_i, \tau_{ij}, \tau_{||ij})$ are calculated by self-self consistent Schrödinger-Poisson algorithm [2] which offers realistic values as this model provides high degree of accuraccy for modelling transport in THz QCLs. The input field $E(z=0,t)$ can take an arbitrary function, and we analysed secant signal for ideal two level atom and experimental pulse for superlattice quantum system.

References

[1] R.W. Ziolkowski *et.al*, PRA 52, 4, 3082 (1995).
[2] A. Demić *et.al*, AIP Advances 9 no. 9, 095019 (2019).

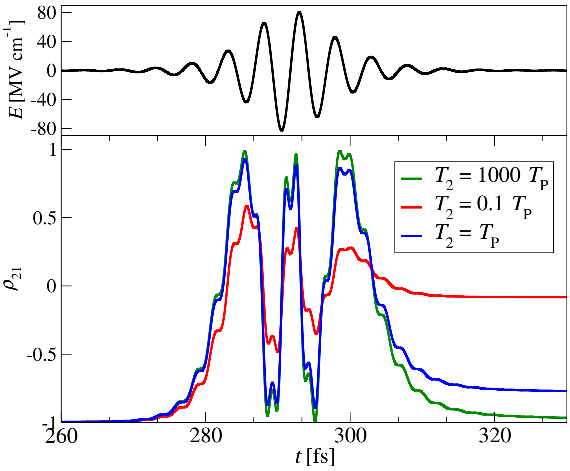

Fig. 1: Top: Secant signal $E_0^{6\pi} = 63$ MV cm^{-1} that analytically causes three Rabi SIT flops of an ideal two level atom from [1]. Bottom: Population inversion when dephasing time (T_2) is larger, equal and smaller than signal width T_P.

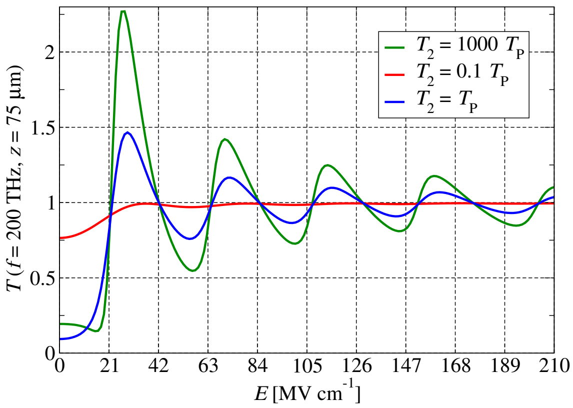

Fig. 2: Transmittance at resonant frequency (200 THz) of ideal two level atom from [1] for different electrical field amplitudes when dephasing time (T_2) is larger, equal and smaller than signal width T_P. Oscillations at $E_0^{2\pi} = 21$ MV cm^{-1} can be clearly observed.

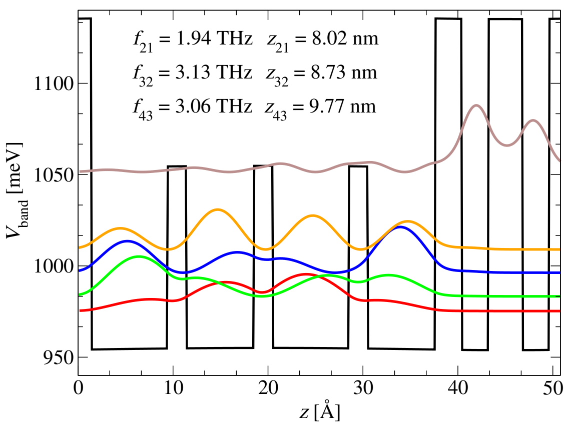

Fig. 3: Bandstructure of superlattice GaAs/Al$_\mathrm{x}$Ga$_{1-\mathrm{x}}$As. designed for absorption at 1.94 THz. Conduction band profile and wafefunction moduii squared are presented. The layer structure is **28**/80/**20**/71/**20**/80/**20**/71/**28**/*28*/**36**/*28*, barriers are shown in bold text (x=0.11 and 0.2) and underlined wells are doped by 2 cm^{-1}.

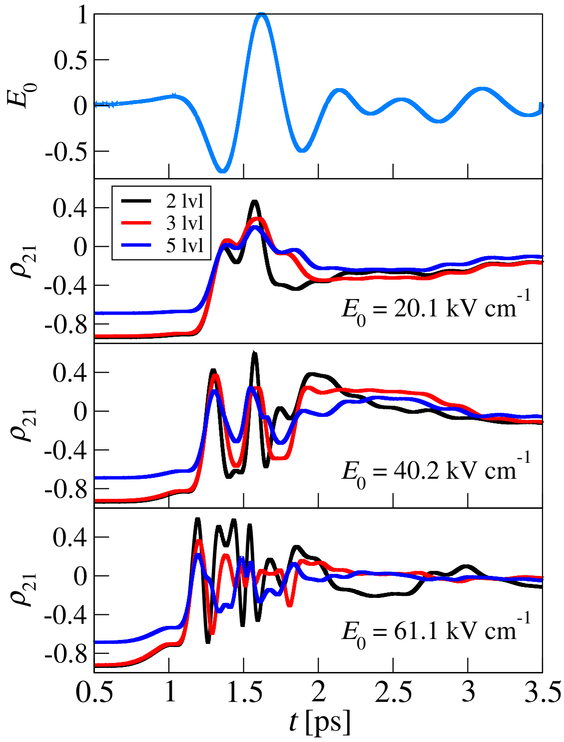

Fig. 4: Top: Experimental input signal generated at resonance of $\sim$ 2 THz presented with normalised amplitude that can be varied experimentally and within the model. Bottom: Population inversion at different input field amplitudes when 2,3 and 5 levels are considered in model in Eqs. (1) and (2). Quantum system parameters are calculated for the structure in Fig. 3, displaying dephasing time $\tau_{||21} = 0.6$ ps.

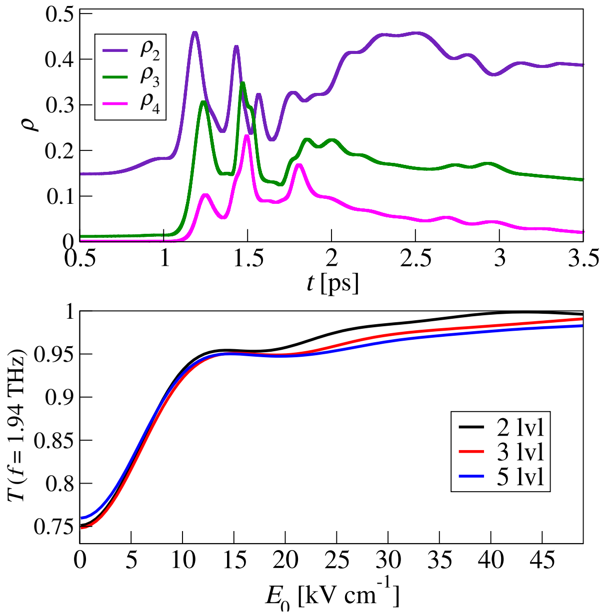

Fig. 5: Top: Populations (in %) at input electric field $E_0 = 40.2$ kV cm^{-1} when five levels are considered in the model. As the signal has spectral contributions $\sim$ 3 THz and structure's dipole elements are significantly large, complex mulitphoton absorption is observed. This, combined with short dephasing time is responsible for the incomplete Rabi oscillations in Fig. 4. Bottom: Transmittance at resonant frequency (1.94 THz) for different electrical field amplitudes when 2,3 and 5 levels are considered in the model.